STAT/MA 41600
In-Class Problem Set #42: December 2, 2016
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Problem Set 42 Answers

1a. The density of $X_{(1)}$ is $f_{X_{(1)}}(x) = \binom{3}{0,1,2}(x/18)(x^2/36)^0(1-x^2/36)^2 = x/6-x^3/108+x^5/7776$ for $0 < x < 6$.
1b. The expected value is $\mathbb{E}(X_{(1)}) = \int_0^6 (x)(x/6 - x^3/108 + x^5/7776)\,dx = (x^3/18 - x^5/540 + x^7/54432)|_{x=0}^{6} = 96/35$.
1c. The density of $X_{(2)}$ is $f_{X_{(2)}}(x) = \binom{3}{1,1,1}(x/18)(x^2/36)^1(1-x^2/36)^1 = x^3/108 - x^5/3888$ for $0 < x < 6$.
1d. The expected value is $\mathbb{E}(X_{(2)}) = \int_0^6 (x)(x^3/108 - x^5/3888)\,dx = (x^5/540 - x^7/27216)|_{x=0}^{6} = 144/35$.

2a. The density of $X_{(3)}$ is $f_{X_{(3)}}(x) = \binom{3}{2,1,0}(x/18)(x^2/36)^2(1-x^2/36)^0 = x^5/7776$ for $0 < x < 6$.
2b. The expected value is $\mathbb{E}(X_{(3)}) = \int_0^6 (x)(x^5/7776)\,dx = x^7/54432|_{x=0}^{6} = 36/7$.
2c. We verify that $\mathbb{E}(X_{(1)}) + \mathbb{E}(X_{(2)}) + \mathbb{E}(X_{(3)}) = 96/35 + 144/35 + 36/7 = 12$.

3a. The pdf of $U_{(3)}$ is $f_{U_{(3)}}(u) = \binom{7}{2,1,4}(1/5)(u/5)^2(1-u/5)^4 = 21(u^2/25 - 4u^3/125 + 6u^4/625 - 4u^5/3125 + u^6/15625)$.
3b. The mean of $U_{(3)}$ is $\mathbb{E}(U_{(3)}) = \int_0^5 (u)(21)(u^2/25 - 4u^3/125 + 6u^4/625 - 4u^5/3125 + u^6/15625)\,du = 21\int_0^5 (u^3/25 - 4u^4/125 + 6u^5/625 - 4u^6/3125 + u^7/15625)\,du = 21(u^4/100 - 4u^5/625 + 6u^6/3750 - 4u^7/21875 + u^8/125000)|_{u=0}^{5} = 15/8$.
3c. The mean of $U_{(4)}$ must be halfway between 0 and 5, because all of the U_j's are left-to-right symmetric, i.e., their densities are balanced around 5/2. So we must have $\mathbb{E}(U_{(4)}) = 5/2$.

4a. Since X is a geometric random variable with $p = 1/5$ and $q = 4/5$, then $\mathrm{Var}(X) = q/p^2 = (4/5)/(1/5)^2 = 20$.
4b. Since X is discrete uniform random variable with $N = 5$, then $\mathrm{Var}(X) = \frac{5^2-1}{12} = 2$.