STAT/MA 41600
In-Class Problem Set #40: November 28, 2016

1. Suppose that X and Y have joint probability mass function

$$p_{X,Y}(x,y) = (3/2)(1/2)^x(1/2)^y \qquad \text{for integers } 1 \le x \le y,$$

and $p_{X,Y}(x,y) = 0$ otherwise.
1a. Find the probability mass function of X.
1b. Find the conditional probability mass function $p_{Y|X}(y \mid x)$ of Y, given $X = x$.
1c. Check that $p_{Y|X}(y \mid x)$ is a valid probability mass function, i.e., that it is nonnegative and sums to 1, i.e., check that $\sum_{y=x}^{\infty} p_{Y|X}(y \mid x) = 1$.

2. Continue with the setup from question 1.
2a. Find $\mathbb{E}(Y \mid X = x)$, for a fixed $x \ge 1$.
2b. Find $\mathbb{E}(Y \mid X = 5)$.

Note: since $Y \ge X$ in this problem, then when $X = 5$, we have $Y \ge 5$. Therefore the value of $\mathbb{E}(Y \mid X = 5)$ must be at least 5.

3. Suppose that X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{12}(4 - xy) & \text{if } 0 < x < 2 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

3a. Find the probability density function $f_X(x)$ of X.
3b. Use your solution to **3a** to find $f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$ for fixed $0 < x < 2$.
3c. Use your solution to **3b** to find $\mathbb{E}(Y \mid X = x) = \int_0^2 y f_{Y|X}(y \mid x)\, dy$, for a fixed $0 < x < 2$.
3d. Use your solution to **3c** to find $\mathbb{E}(Y) = \int_0^2 \mathbb{E}(Y \mid X = x) f_X(x)\, dx$.

[Hint: The focus of this question is 3c. Nonetheless, you can check your solution to 3d by consider $\mathbb{E}(Y)$ from problem set 39.]

4. Consider a pair of random variables X and Y with joint probability density function $f_{X,Y}(x,y) = \frac{1}{8}xy$ for x, y in the triangle where $0 < x < 2$ and $0 < y < 2x$, and $f_{X,Y}(x,y) = 0$ otherwise.

In Problem Set 39, question 4, we showed that if $0 < x < 2$, then $f_{Y|X}(y \mid x) = y/(2x^2)$ for $0 < y < 2x$, and $f_{Y|X}(y \mid x) = 0$ otherwise.

Assume that $0 < x < 2$ is fixed. Find $\mathbb{E}(Y \mid X = x)$.