STAT/MA 41600
In-Class Problem Set #39 part 2: November 21, 2016

1. Suppose that X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{12}(4-xy) & \text{if } 0 < x < 2 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the correlation $\rho(X,Y)$ between X and Y. You are welcome to use the fact that $\text{Cov}(X,Y) = -4/81$, as discovered on Friday.

2. A bag contains 10 red bears, 10 green bears, and 10 blue bears. Ten children pick 3 bears each, without replacement. Let X denote the number of children who get 3 differently colored bears, i.e., one bear of each color. Find $\text{Var}(X)$.

3. A standard deck of 52 cards has 4 Queens. Alice picks 5 cards and Bob picks 5 cards (all without replacement) at random from a shuffled deck of 52 cards. Let X denote the number of Queens that Alice chooses. Let Y denote the number of Queens that Bob chooses. Find the covariance of X and Y.

4. Consider a pair of random variables X and Y with joint probability density function $f_{X,Y}(x,y) = \frac{1}{8}xy$ for x, y in the triangle where $0 < x < 2$ and $0 < y < 2x$, and $f_{X,Y}(x,y) = 0$ otherwise.
4a. Find $\text{Cov}(X,Y)$.
4b. Find $\rho(X,Y)$.