STAT/MA 41600
In-Class Problem Set #37: November 11, 2016
Solutions by Mark Daniel Ward

Problem Set 37 Answers

1a. The probability is $\sum_{j=4200}^{5200} \binom{5200}{j} (8/10)^j (2/10)^{5200-j}$.
1b. Let X denote the number of attendees. Then $P(X \geq 4200) = P(X \geq 4199.5) = P\big(\frac{X-(5200)(.80)}{\sqrt{(5200)(.80)(.20)}} \geq \frac{4199.5-(5200)(.80)}{\sqrt{(5200)(.80)(.20)}}\big) \approx P(Z \geq 1.37) = 1 - P(Z \leq 1.37) = 1 - 0.9147 = 0.0853$.

2. Let X denote a Gamma random variable with $r = 1200$ and $\lambda = 1/3$. Then $P(X > 3500) = P\big(\frac{X-(1200)/(1/3)}{\sqrt{(1200)/(1/3)^2}} > \frac{3500-(1200)/(1/3)}{\sqrt{(1200)/(1/3)^2}}\big) \approx P(Z > -0.96) = P(Z < 0.96) = 0.8315$.

3a. The random variable X is a Poisson random variable with parameter $\lambda = (2)(24) = 48$.
3b. We compute $P(X \geq 45) = P(X \geq 44.5) = P\big(\frac{X-48}{\sqrt{48}} \geq \frac{44.5-48}{\sqrt{48}}\big) \approx P(Z \geq -0.51) = P(Z \leq 0.51) = 0.6950$.

4a. The exact probability is $\sum_{j=20800}^{20900} \frac{e^{-\lambda}\lambda^j}{j!}$ where $\lambda = 1250000/60 = 20833.33$.
4b. Let X denote the number of jelly beans produced during the next 1 minute. Then $P(20800 \leq X \leq 20900) = P(20799.5 \leq X \leq 20900.5) = P\big(\frac{20799.5-20833.33}{\sqrt{20833.33}} \leq \frac{X-20833.33}{\sqrt{20833.33}} \leq \frac{20900.5-20833.33}{\sqrt{20833.33}}\big) \approx P(-0.23 \leq Z \leq 0.47) = P(Z \leq 0.47) - P(Z \leq -0.23) = P(Z \leq 0.47) - P(Z \geq 0.23) = P(Z \leq 0.47) - (1 - P(Z \leq 0.23)) = 0.6808 - (1 - 0.5910) = 0.2718$.