Problem Set 35 Answers

1a. We have $P(17 < X < 18) = P(\frac{17-17.2}{1.8} < \frac{X-17.2}{1.8} < \frac{18-17.2}{1.8}) = P(-0.11 < Z < 0.44) = P(Z < 0.44) - P(Z < -0.11)$. So we get $P(17 < X < 18) = P(Z < 0.44) - P(Z > 0.11) = P(Z < 0.44) - (1 - P(Z < 0.11)) = 0.6700 - (1 - 0.5438) = 0.2138$.

1b. We compute $P(|X - 17.2| > 1) = P(|\frac{X-17.2}{1.8}| > \frac{1}{1.8}) = P(|Z| > 0.56) = P(Z > 0.56) + P(Z < -0.56) = 2P(Z > 0.56) = 2(1 - P(Z < 0.56)) = 2(1 - 0.7123) = 0.5754$.

1c. We have $P(\frac{X-17.2}{1.8} < 2) = P(Z < 2) = 0.9772$.

2a. We have $P(X > 19 \mid X > 18) = \frac{P(X>19 \ \& \ X>18)}{P(X>18)} = \frac{P(X>19)}{P(X>18)}$. We have $P(X > 19) = P(\frac{X-17.2}{1.8} > \frac{19-17.2}{1.8}) = P(Z > 1) = 1 - P(Z < 1) = 1 - 0.8413 = 0.1587$, and $P(X > 18) = P(\frac{X-17.2}{1.8} > \frac{18-17.2}{1.8}) = P(Z > 0.44) = 1 - P(Z < 0.44) = 1 - 0.6700 = 0.3300$. So we conclude that $P(X > 19 \mid X > 18) = 0.1587/0.3300 = 0.4809$.

2b. We have $P(X < 19 \mid X < 20) = \frac{P(X<19 \ \& \ X<20)}{P(X<20)} = \frac{P(X<19)}{P(X<20)}$. We have $P(X < 19) = P(\frac{X-17.2}{1.8} < \frac{19-17.2}{1.8}) = P(Z < 1) = 0.8413$, and $P(X < 20) = P(\frac{X-17.2}{1.8} < \frac{20-17.2}{1.8}) = P(Z < 1.56) = 0.9406$. So we conclude that $P(X < 19 \mid X < 20) = 0.8413/0.9406 = 0.8944$.

2c. We have $0.40 = P(17.2 - c < X < 17.2 + c) = P(\frac{17.2-c-17.2}{1.8} < \frac{X-17.2}{1.8} < \frac{17.2+c-17.2}{1.8}) = P(-c/1.8 < Z < c/1.8) = P(Z < c/1.8) - P(Z < -c/1.8) = P(Z < c/1.8) - P(Z > c/1.8) = P(Z < c/1.8) - (1 - P(Z < c/1.8)) = 2P(Z < c/1.8) - 1$. Thus $2P(Z < c/1.8) = 1.40$, so $P(Z < c/1.8) = 0.70$. So $c/1.8 = 0.52$, and thus $c = 0.936$.

3a. The random variable Y is Binomial with $n = 10$ and $p = P(X > 19) = 0.1587$, as in question 2a.

3b. We have $P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - P(Y = 0) - P(Y = 1) - P(Y = 2) = 1 - \binom{10}{0}p^0(1-p)^{10} - \binom{10}{1}p^1(1-p)^9 - \binom{10}{2}p^2(1-p)^8 = 0.2029$.

4a. We have $P(X > 0) = P(\frac{X-5}{\sqrt{2}} > \frac{0-5}{\sqrt{2}}) = P(Z > -3.54) = P(Z < 3.54) = 0.9998$.

4b. We have $P(Y > 0) = P(\frac{1}{3}X - 2 > 0) = P(X > 6) = P(\frac{X-5}{\sqrt{2}} > \frac{6-5}{\sqrt{2}}) = P(Z > 0.71) = 1 - P(Z < 0.71) = 1 - 0.7611 = 0.2389$.