$\label{eq:stat} \begin{array}{c} {\rm STAT/MA~41600} \\ {\rm In-Class~Problem~Set~\#34:~November~4,~2016} \end{array}$

1. Consider a Beta random variable X with parameters $\alpha = 3$ and $\beta = 2$.

1a. Is it more likely that X is smaller than 1/2 or larger than 1/2?

1b. Verify directly, without using the general formula $\mathbb{E}(X) = \alpha/(\alpha + \beta)$, that the expected value of X is $\mathbb{E}(X) = 3/5$.

2. Same setup as question #1.

2a. Find $P(X > 1/2 \mid X > 1/4)$. [Hint: You can use your work from 1a to help with 2a.] **2b.** Find the probability that the distance between X and 1/2 is at least 2/5, i.e., find P(|X - 1/2| > 2/5).

3. Same setup as question #1. If U is a continuous uniform random variable on (0, 1) and U is independent from X, find P(U < X).

4. Review question: Roll a die until the first 5 appears. Let X denote the number of rolls needed (including the 5 itself). Find the probability that X is an even number.