STAT/MA 41600
In-Class Problem Set #33: November 2, 2016
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Problem Set 33 Answers

1a. No, $X + Y$ is not a Gamma random variable, because X and Y are not independent, and moreover, Y is not an exponential random variable.
1b. Yes, $V + W$ is a Gamma random variable with parameters $r = 2$ and $\lambda = 1/2$.
1c. Yes, $X_1 + \cdots + X_n$ is a Gamma random variable with parameters $r = n$ and λ.

2a. No, $X + Y$ is not a Gamma random variable, because the λ values of X and Y are different. The λ for X is $1/2$ and the λ for Y is $2/3$.
2b. Since X and Y are independent exponential random variables, the variance of $X + Y$ is $2^2 + (3/2)^2 = 25/4$. So the standard deviation of $X + Y$ is $\sqrt{25/4} = 5/2$.

3. Since X and Y are independent exponential random variables, each with $\lambda = 4$, then $X + Y$ is a Gamma random variable with parameters $r = 2$ and $\lambda = 4$. Thus, we get $P(X + Y \leq 1) = \int_0^1 \frac{4^2}{\Gamma(2)} x^{2-1} e^{-4x}\, dx = \int_0^1 16xe^{-4x}\, dx = 1 - 5e^{-4}$.

Alternatively, we could compute $P(X + Y \leq 1) = \int_0^1 \int_0^{1-x} 16e^{-4x-4y}\, dy\, dx = 1 - 5e^{-4}$.

4. A geometric random variable with parameter p has expected value $1/p$. So, if we ignore the win itself, the geometric number of losses has expected value $1/p - 1 = 1/p - p/p = (1 - p)/p = q/p$. So the expected value is $e^{-3}/(1 - e^{-3}) = 0.0498/0.9502 = 0.0524$.