STAT/MA 41600
In-Class Problem Set #32 part 2: October 31, 2016

1. Let X denote the time (in minutes) until the full moon is visible from behind the clouds, and Y denote the time (in minutes) until the next wolf howls. Suppose X and Y have joint probability density function $f_{X,Y}(x,y) = 15e^{-2x-3y}$ for $y > x$, and $f_{X,Y}(x,y) = 0$ otherwise. (Note that X and Y are dependent. The wolf will not howl until after the moon appears, and hence $Y > X$.)
1a. What is the probability density function $f_X(x)$ of X?
1b. What is the probability density function $f_Y(y)$ of Y?

2. Suppose that you will hear exactly 1 scary howl within the next 1 minute, and the time U when you hear this howl is uniformly distributed in the interval $(0,1)$. Let $X = -3\ln U$. What kind of random variable is X? What is the expected value of X?

3. While trick-or-treating, let V and W be the (respective) times until you see the next vampire and werewolf. Assume that V and W are independent exponential random variables, each with parameter $1/2$, i.e., each with expected value 2. Find $\mathbb{E}(\max(V,W))$.

4a. Consider two pumpkins whose lights have independent lifetimes, each of which is exponentially distributed, with parameter λ, i.e., expected value $1/\lambda$. What is the expected value of the first time that one of the pumpkin lights go out?
4b. Repeat question 4a with three pumpkins.
4c. Repeat question 4a with n pumpkins.
Hint: This question is easier than question #3. No integrals are needed for question #4.