STAT/MA 41600
In-Class Problem Set #32: October 28, 2016
Solutions by Mark Daniel Ward

Problem Set 32 Answers

1. We have $P(X > U) = \int_0^5 \int_u^\infty (\frac{1}{5})(\frac{1}{2}e^{-x/2})\,dx\,du = \int_0^5 (\frac{1}{5})(e^{-u/2})\,du = -\frac{2}{5}e^{-u/2}|_{u=0}^5 = \frac{2}{5} - \frac{2}{5}e^{-5/2}$.

2a. We compute $P(X > 1.6) = \int_{1.6}^\infty \frac{1}{2.8}e^{-(1/2.8)x}\,dx = e^{-1.6/2.8} = 0.5647$.

2b. We compute $P(X > 3.5 \mid X > 1.1) = \frac{P(X>3.5\ \&\ X>1.1)}{P(X>1.1)} = \frac{P(X>3.5)}{P(X>1.1)} = \frac{\int_{3.5}^\infty \frac{1}{2.8}e^{-(1/2.8)x}\,dx}{\int_{1.1}^\infty \frac{1}{2.8}e^{-(1/2.8)x}\,dx} = \frac{e^{-3.5/2.8}}{e^{-1.1/2.8}} = e^{-2.4/2.8} = 0.4244$. Instead, we could have used the memoryless property of exponential random variables, to get $P(X > 3.5 \mid X > 1.1) = P(X > 2.4) = e^{-2.4/2.8} = 0.4244$.

3. [On the problem set, I originally wrote 15 seconds = 4 minutes, but of course I meant to write 15 seconds = 1/4 minute = 0.25 minutes.] The probability that the next bird is blue is $\int_0^\infty \int_x^\infty \frac{1}{15}e^{-x/15}\frac{1}{20}e^{-y/20}\,dy\,dx = \int_0^\infty \frac{1}{15}e^{-x/15}e^{-x/20}\,dx = \int_0^\infty (\frac{1}{15})e^{-7x/60}\,dx = (\frac{1}{15})\frac{e^{-7x/60}}{-7/60}|_{x=0}^\infty = (\frac{1}{15})/(\frac{7}{60}) = 4/7$.

Alternatively, solving the problem with minutes instead of seconds, we get the same answer: $\int_0^\infty \int_x^\infty 4e^{-4x}3e^{-3y}\,dy\,dx = \int_0^\infty 4e^{-4x}e^{-3x}\,dx = \int_0^\infty 4e^{-7x}\,dx = -(4/7)e^{-7x}|_{x=0}^\infty = 4/7$.

4a. Yes, if X is an exponential random variable, then cX is an exponential random variable too. If X has parameter λ, then $P(cX > a) = P(X > a/c) = e^{-\lambda(a/c)} = e^{-(\lambda/c)a}$, so λ/c is the parameter of cX. Another way to see this is to note that $\mathbb{E}(X) = 1/\lambda$, so $\mathbb{E}(cX) = c/\lambda$, so that λ/c is the parameter of cX.

4b. No, it is not the case that $cX + d$ is an exponential random variable. Since X is always positive, then $cX + d$ is always bigger than d, i.e., $cX + d$ takes values in the range (d, ∞), but exponential random variables take values in the range $(0, \infty)$. So $cX + d$ is not an exponential random variable.