STAT/MA 41600 In-Class Problem Set #32: October 28, 2016

1. If X is an exponential random variable with $\mathbb{E}(X) = 2$, and U is a continuous uniform random variable on the interval [0, 5], what is the probability that X is bigger than U?

2. As a rainstorm is ending, raindrops are landing only occasionally on a window. Let X denote the time (in seconds) until the next raindrop hits the window. Assume that X is exponential, with expected value of 2.8 seconds.

2a. Find P(X > 1.6).
2b. Find P(X > 3.5 | X > 1.1).

3. Suppose that birds flutter past a window every once in awhile. Assume the time between arrivals of red birds is exponential, with average waiting time of 20 seconds = 0.3333 minutes between arrivals. Assume that the time between arrivals of blue birds is exponential, with average waiting time of 15 seconds = 0.25 minutes between arrivals. Also assume that the arrivals of the red birds and blue birds are independent. If we start looking for the next bird, what is the probability that it will be blue?

4a. If X is an exponential random variable with parameter $\lambda > 0$, and if c is a positive constant, is it always the case that cX is an exponential random variable too? If yes, then what is the parameter of cX? If no, then why not?

4b. If X is an exponential random variable with parameter $\lambda > 0$, and if c and d are positive constants, is it always the case that cX + d is an exponential random variable too? If yes, then what is the parameter of cX + d? If no, then why not?