STAT/MA 41600
In-Class Problem Set #27: October 19, 2016

1. Suppose that X and Y have joint probability density function $f_{X,Y}(x,y) = 15e^{-5x-3y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.
1a. Is it true in this setup that $P(Y > 3/10 \mid X > 1/10) = P(Y > 3/10)$? Why or why not?
1b. Calculate the value of $P(Y > 3/10 \mid X > 1/10)$.

2. Suppose that X and Y have joint density $f_{X,Y}(x,y) = 24e^{-5x-3y}$ for $y > x > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.
2a. Calculate $P(Y > 3/10 \mid X > 1/10)$. You may use the fact that $P(X > 1/10) = e^{-8/10}$, as calculated in question 3 of the previous problem set.
2b. Find $f_X(x)$, and use this to find $f_{Y|X}(y \mid x)$.
2c. Calculate $P(Y > 3/10 \mid X = 1/10)$.

3. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0,0)$, $(8,0)$, and $(0,4)$.
3a. Find $f_{Y|X}(y \mid 2)$.
3b. Find $P(Y > 1 \mid X = 2)$.
3c. Find $P(Y > 1 \mid X > 2)$.

4. Suppose that X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{12}(4 - xy) & \text{if } 0 < x < 2 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

4a. Find $f_{Y|X}(y \mid x)$. You are welcome to use $f_X(x)$, as calculated in question 4b of the previous problem set.
4b. Calculate $P(Y < 4/3 \mid X = 2/3)$.
4c. Calculate $P(Y < 4/3 \mid X < 2/3)$.