STAT/MA 41600
In-Class Problem Set #25: October 14, 2016
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Problem Set 25 Answers

1. We compute $P(Y > X/2) = \int_0^\infty \int_{x/2}^\infty 15e^{-5x-3y}\,dy\,dx = \int_0^\infty -5e^{-5x-3y}|_{y=x/2}^\infty\,dx = \int_0^\infty 5e^{-5x-3x/2}\,dx = \int_0^\infty 5e^{-13x/2}\,dx = -(10/13)e^{-13x/2}|_{x=0}^\infty = 10/13$.

2a. We compute $P(\max(X,Y) \le 1) = \int_0^1 \int_0^1 15e^{-5x-3y}\,dy\,dx = \int_0^1 -5e^{-5x-3y}|_{y=0}^1\,dx = \int_0^1 5e^{-5x}(1-e^{-3})\,dx = -e^{-5x}|_{x=0}^1(1-e^{-3}) = (1-e^{-5})(1-e^{-3})$.

2b. We compute $P(1 \le \min(X,Y)) = \int_1^\infty \int_1^\infty 15e^{-5x-3y}\,dy\,dx = \int_1^\infty -5e^{-5x-3y}|_{y=1}^\infty\,dx = \int_1^\infty 5e^{-5x-3}\,dx = -e^{-5x-3}|_{x=1}^\infty = e^{-8}$.

3. We find the density for X by integrating over all of the relevant y values. So we compute $f_X(x) = \int_x^\infty 24e^{-5x-3y}\,dy = -8e^{-5x-3y}|_{y=x}^\infty = 8e^{-5x-3x} = 8e^{-8x}$ for $x > 0$, and $f_X(x) = 0$ otherwise.

4. We have $P(X+Y \le 4) = \int_0^4 \int_0^{4-x} \frac{1}{64}(4-x)(4-y)\,dy\,dx = \int_0^4 \frac{1}{64}(4-x)(4y-y^2/2)|_{y=0}^{4-x}\,dx = \int_0^4 \frac{1}{64}(4-x)(4(4-x)-(4-x)^2/2)\,dx = \int_0^4 \frac{1}{64}(4-x)(8-x^2/2)\,dx = \int_0^4 \frac{1}{64}(32-8x-2x^2+x^3/2)\,dx = \frac{1}{64}(32x-4x^2-(2/3)x^3+x^4/8)|_{x=0}^4 = 5/6$.