STAT/MA 41600
In-Class Problem Set #24: October 12, 2016
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Problem Set 24 Answers

1a. We need $1 = \int_0^5 (k)(x)(5-x)\,dx = k \int_0^5 (5x - x^2)\,dx = k((5/2)x^2 - x^3/3)|_{x=0}^5 = (k)(125/6)$, and thus $k = 6/125$.

1b. We compute $P(X > 4 \text{ or } X < 1) = \int_0^1 (6/125)(x)(5-x)\,dx + \int_4^5 (6/125)(x)(5-x)\,dx = (6/125)((5/2)x^2 - x^3/3)|_{x=0}^1 + (6/125)((5/2)x^2 - x^3/3)|_{x=4}^5 = 13/125 + 13/125 = 26/125$.

2. We have $P(|Y - 3| \leq 1) = P(2 \leq Y \leq 4) = \int_2^4 (2/7)e^{-(2/7)(y)}\,dy = -e^{-(2/7)(y)}|_{y=2}^4 = e^{-(2/7)(2)} - e^{-(2/7)(4)} = e^{-4/7} - e^{-8/7} = 0.2458$.

3a. We have $f_X(x) = \frac{d}{dx}(1 - e^{-5x}) = 5e^{-5x}$ for $x > 0$, and $f_X(x) = \frac{d}{dx}(0) = 0$ otherwise.

3b. We have $P(1/4 < X < 1/3) = P(X < 1/3) - P(X \leq 1/4) = F_X(1/3) - F_X(1/4) = (1 - e^{-5(1/3)}) - (1 - e^{-5(1/4)}) = e^{-5/4} - e^{-5/3} = 0.0976$.

3c. We compute $P(1/4 < X < 1/3) = \int_{1/4}^{1/3} 5e^{-5x}\,dx = -e^{-5x}|_{x=1/4}^{1/3} = e^{-5(1/4)} - e^{-5(1/3)} = e^{-5/4} - e^{-5/3} = 0.0976$.

4. The pdf of X is $1/70$; to see this, note $1/70$ is the only constant such that $\int_{30}^{100} 1/70\,dx = 1$. Thus, we get $P(80 < 2X < 164) = P(40 < X < 82) = \int_{40}^{82} 1/70\,dx = 42/70 = 3/5$.