STAT/MA 41600 In-Class Problem Set #19: September 30, 2016

1. Consider a standard deck of 52 cards. Deal 5 cards in a row, left to right, on the table. What is the variance of the number of Jacks that are dealt?

2. Consider a collection of 3 red bears, 3 green bears, and 3 blue bears, all placed into a bag. Draw 5 bears from the bag of bears (without replacement).

2a. What is the variance of the number of green bears that are chosen?

2b. What is the variance of the number of green or blue bears that are chosen?

2c. What is the variance of the number of green bears that are not chosen, i.e., that remain in the bag after the selection of 5 bears is over?

3. Fix some values n, M, N. Let X be a hypergeometric random variable with parameters n, M, N. Let Y be a Binomial random variable with parameters n and p = M/N.

3a. Are $\mathbb{E}(X)$ and $\mathbb{E}(Y)$ always the same?

3b. Is it always the case that $Var(Y) \ge Var(X)$?

4. Consider a company that manufactures 50,000 toys in a year. Suppose that 500 out of the 50,000 toys were defective. (You may assume that the manufacturing process is independent, i.e., the defective toys do not affect the manufacturing of other toys.) An inspector checks 200 of these 50,000 toys. Let X denote the number of defective toys checked by the inspector. 4a. What is the probability mass function of X?

4b. Give an exact expression for P(X = 4). You do not need to evaluate the expression.

4c. Give an approximate expression for P(X = 4), and evaluate the approximate expression.