STAT/MA 41600
In-Class Problem Set #18: September 28, 2016
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Problem Set 18 Answers

1a. During the next 3 minutes, the expected number of people who get food at the salad bar is $(3)(2) = 6$. The probability that at least 4 people get food at the salad bar during the next 3 minutes is $\sum_{x=4}^{\infty} \frac{e^{-6}6^x}{x!} = 1 - \sum_{x=0}^{3} \frac{e^{-6}6^x}{x!} = 1 - e^{-6}(1+6+6^2/2+6^3/6) = 1 - 61e^{-6} = 0.8488$.
1b. During the next 90 seconds, the expected number of people who get food at the salad bar is $(1.5)(2) = 3$. The probability that at least 3 people get food at the salad bar during the next 90 seconds is $\sum_{x=3}^{\infty} \frac{e^{-3}3^x}{x!} = 1 - \sum_{x=0}^{2} \frac{e^{-3}3^x}{x!} = 1 - e^{-3}(1+3+3^2/2) = 1 - (17/2)e^{-3} = 0.5768$.
1c. Poisson random variables have the same mean and expected value. The expected value is $(5)(2) = 10$, so the variance is 10 too.

2. The number of people struck by lightning among these 500,000 people is Binomial with $n = 500{,}000$ and $p = 1/1{,}042{,}000$. So the probability that at least one of them is struck by lightning is $\sum_{x=1}^{500{,}000} \binom{500{,}000}{x} (\frac{1}{1{,}042{,}000})^x (\frac{1{,}041{,}999}{1{,}042{,}000})^{500{,}000-x}$, or equivalently $1 - \binom{500{,}000}{0} (\frac{1}{1{,}042{,}000})^0 (\frac{1{,}041{,}999}{1{,}042{,}000})^{500{,}000-0} = 1 - (1{,}041{,}999/1{,}042{,}000)^{500{,}000} = 0.3811$.

Alternatively: The number of people struck by lightning is approximately Poisson with $\lambda = (500{,}000)(1/1{,}042{,}000)$. So the probability that at least one of them is struck by lightning is $\sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = 1 - \frac{e^{-\lambda}\lambda^0}{0!} = 1 - e^{-\lambda} = 1 - e^{-(500{,}000)(1/1{,}042{,}000)} = 0.3811$.

3a. Since X_1, X_2, X_3 are independent Poisson random variables, each with mean 0.8, then $X_1 + X_2 + X_3$ is also a Poisson random variable with mean 2.4, so $P(X_1 + X_2 + X_3 \leq 3) = \sum_{x=0}^{3} \frac{e^{-2.4}(2.4)^x}{x!} = e^{-2.4}(1 + 2.4 + (2.4)^2/2 + (2.4)^3/6) = e^{-2.4}(8.584) = 0.7787$.
3b. Since Y is a Poisson random variable with $\lambda = 2.4$, then $p_Y(0) = e^{-2.4}(2.4)^0/0! = 0.0907$, $p_Y(1) = e^{-2.4}(2.4)^1/1! = 0.2177$, $p_Y(2) = e^{-2.4}(2.4)^2/2! = 0.2613$, $p_Y(3) = e^{-2.4}(2.4)^3/3! = 0.2090$, $p_Y(4) = e^{-2.4}(2.4)^4/4! = 0.1254$, ..., so $p_Y(y)$ is largest when $y = 2$.

4. We see that

$$\begin{aligned}
\mathbb{E}((X)(X-1)(X-2)) &= \sum_{x=0}^{\infty} (x)(x-1)(x-2)\frac{e^{-\lambda}\lambda^x}{x!} \\
&= e^{-\lambda} \sum_{x=3}^{\infty} (x)(x-1)(x-2)\frac{\lambda^x}{x!} \\
&= e^{-\lambda} \sum_{x=3}^{\infty} \frac{\lambda^x}{(x-3)!} \\
&= \lambda^3 e^{-\lambda} \sum_{x=3}^{\infty} \frac{\lambda^{x-3}}{(x-3)!} \\
&= \lambda^3 e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} \\
&= \lambda^3 e^{-\lambda} e^{\lambda} = \lambda^3 = 5^3 = 125.
\end{aligned}$$