STAT/MA 41600
In-Class Problem Set #11: September 16, 2016

1. Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. These 6 bears are all placed around a circular table with 6 chairs, and all arrangements are equally likely. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs.

Define some indicator random variables X_1, X_2, X_3 so that $X = X_1 + X_2 + X_3$, or perhaps $X_1, \ldots, X_6$ so that $X = X_1 + \cdots + X_6$. (There are several ways that you could accomplish this.) Then use the random variables you created to find $\mathbb{E}(X)$.

2a. Pick two cards simultaneously at random from a well-shuffled deck of 52 cards. There are 36 cards which have numbers on them (cards 2 through 10, in each of the 4 suits), and there are 16 cards without numbers on them (A, J, Q, K, in each of the 4 suits). Let X be the number of cards that you get with numbers on them.

Define some indicator random variables X_1, X_2 so that $X = X_1 + X_2$, or perhaps $X_1, \ldots, X_{36}$ so that $X = X_1 + \cdots + X_{36}$. Then use the random variables you created to find $\mathbb{E}(X)$.

2b. Reconsider question **2a**, but this time pick 3 cards.

Define some indicator random variables X_1, X_2, X_3 so that $X = X_1 + X_2 + X_3$, or perhaps $X_1, \ldots, X_{36}$ so that $X = X_1 + \cdots + X_{36}$. Then use the random variables you created to find $\mathbb{E}(X)$.

3a. Consider a deck of 5 cards with the values A, 2, 3, 4, 5. We deal one card at a time from this deck of 5 cards, with replacement of the card back into the deck—and also shuffling—in between each deal. We continue in this fashion until the first A appears, and then we stop afterwards. Let X be the number of cards dealt.

Define some indicator random variables $X_1, X_2, \ldots$ so that $X = X_1 + X_2 + \cdots = \sum_{j=1}^{\infty} X_j$. Then use the random variables you created to find $\mathbb{E}(X)$.

3b. Reconsider question **3a**, but this time do not replace the cards after they are dealt.

Define some indicator random variables $X_1, \ldots, X_5$ so that $X = X_1 + \cdots + X_5$. Then use the random variables you created to find $\mathbb{E}(X)$.

4. Suppose Alice rolls a 6-sided die, and Bob rolls a 4-sided die. Let X denote the *maximum* value on the two dice.

Define some indicator random variables $X_1, \ldots, X_6$ so that $X = X_1 + \cdots + X_6$. Then use the random variables you created to find $\mathbb{E}(X)$.