STAT/MA 41600 In-Class Problem Set #10: September 14, 2016 Solutions by Mark Daniel Ward

Problem Set 10 Answers

1. We recall from Problem Set #7, question 4, that P(X = 3) = 2/15, P(X = 2) = 1/5, P(X = 1) = 2/5, and P(X = 0) = 4/15. Thus $\mathbb{E}(X) = (3)(2/15) + (2)(1/5) + (1)(2/5) + (0)(4/15) = 6/5$.

2a. We recall from Problem Set #7, question 3, that P(X = 2) = 105/221, P(X = 1) = 96/221, and P(X = 0) = 20/221. Thus $\mathbb{E}(X) = (2)(105/221) + (1)(96/221) + (0)(20/221) = 18/13$.

2b. We compute P(X = 3) = ((36)(35)(34))/((52)(51)(50)) = 21/65, P(X = 2) = 3((36)(35)(16))/((52)(51)(50)) = 504/1105, P(X = 1) = 3((36)(16)(15))/((52)(51)(50)) = 216/1105, and P(X = 0) = ((16)(15)(14))/((52)(51)(50)) = 28/1105. Thus $\mathbb{E}(X) = (3)(21/65) + (2)(504/1105) + (1)(216/1105) + (0)(28/1105) = 27/13$.

3a. We have $P(X = x) = (4/5)^{x-1}(1/5)$ for $x \ge 1$ and P(X = x) = 0 otherwise. Thus we get $\mathbb{E}(X) = \sum_{x=1}^{\infty} (x)(4/5)^{x-1}(1/5) = (1/5) \sum_{x=1}^{\infty} \frac{d}{dq} q^x \Big|_{q=4/5} = (1/5) \frac{d}{dq} \sum_{x=1}^{\infty} q^x \Big|_{q=4/5} = (1/5) \frac{d}{dq} \frac{1}{1-q} \Big|_{q=4/5} = (1/5) \frac{1}{(1-q)^2} \Big|_{q=4/5} = (1/5) \frac{1}{(1-4/5)^2} = 5.$

3b. We have P(X = x) = 1/5 for integers $1 \le x \le 5$, and P(X = x) = 0 otherwise. Therefore we get $\mathbb{E}(X) = (1/5)(1) + (1/5)(2) + (1/5)(3) + (1/5)(4) + (1/5)(5) = 3$.

4a. We have $p_X(6) = 1/6$, $p_X(5) = 1/6$, $p_X(4) = 7/24$, $p_X(3) = 5/24$, $p_X(2) = 3/24$, and $p_X(1) = 1/24$. **4b.** We have $\mathbb{E}(X) = (6)(1/6) + (5)(1/6) + (4)(7/24) + (3)(5/24) + (2)(3/24) + (1)(1/24) = 47/12$.