STAT/MA 41600
In-Class Problem Set #9: September 12, 2016

1. Roll a 4-sided die and a 6-sided die. Let X and Y (respectively) denote the values that appear. What is $P(X = Y)$?

2. Roll a pair of 6-sided dice (one blue dice and one red dice) repeatedly, until the sum of the two dice is 7 or larger. Let X denote the value of the blue die on the final roll, and let Y denote the value of the red die on the final roll.
2a. Are X and Y independent?
2b. Find the values $p_{Y|X}(1 \mid 4)$; $p_{Y|X}(2 \mid 4)$; $p_{Y|X}(3 \mid 4)$; $p_{Y|X}(4 \mid 4)$; $p_{Y|X}(5 \mid 4)$; $p_{Y|X}(6 \mid 4)$.

3. Suppose that X and Y are independent random variables, such that X has mass $p_X(x) = (1/3)(2/3)^{x-1}$ for integers $x \geq 1$, and Y has mass $p_Y(y) = (3/4)(1/4)^{y-1}$ for integers $y \geq 1$.
3a. Find the probability that X and Y are equal.
3b. Find the probability that X is strictly larger than Y.

4. Suppose that X and Y are random variables with joint probability mass function $p_{X,Y}(x, y) = (5/9)(1/2)^{x-1}(1/3)^{y-1}$ for integers $1 \leq x \leq y$.
4a. Find $P(Y > 5 \mid X = 2)$.
4b. Are X and Y dependent or independent?
4c. Find the probability mass function of X.