STAT/MA 41600
In-Class Problem Set #5: September 2, 2016
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Problem Set 5 Answers

1. Let B denote the event that the a red side appears. Let A denote the event that the first die was chosen. We compute $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A\cap B)+P(A^c\cap B)} = \frac{P(A)P(B|A)}{P(A)P(B|A)+P(A^c)P(B|A^c)} = \frac{(1/2)(2/6)}{(1/2)(2/6)+P(1/2)(1/4)} = 4/7$.

2. Let A, I, N denote (respectively) the events that the person owns an Android, iPhone, or no cell phone at all. Let S denote the event that the customer is satisfied.
2a. We have $P(I|S) = \frac{P(I\cap S)}{P(S)} = \frac{P(I\cap S)}{P(I\cap S)+P(A\cap S)+P(N\cap S)} = \frac{P(I)P(S|I)}{P(I)P(S|I)+P(A)P(S|A)+P(N)P(S|N)} = \frac{(0.177)(0.90)}{(0.177)(0.90)+(0.807)(0.70)+(0.016)(0)} = 0.2200$.
2b. We have $P(N|S^c) = \frac{P(N\cap S^c)}{P(S^c)} = \frac{P(I\cap S^c)}{P(I\cap S^c)+P(A\cap S^c)+P(N\cap S^c)} = \frac{P(I)P(S^c|I)}{P(I)P(S^c|I)+P(A)P(S^c|A)+P(N)P(S^c|N)} = \frac{(0.016)(1)}{(0.177)(0.10)+(0.807)(0.30)+(0.016)(1)} = 0.0580$.
2c. An equivalent question is: What is the probability that the first 3 people are not Android customers? So the desired probability is $(1-0.807)^3 = 0.0072$.

3a. Let R denote the event that the M&M is red, and B denotes the probability it is broken. Then $P(R|B) = \frac{P(R\cap B)}{P(B)} = \frac{(.13)(.10)}{(.24)(.10)+(.13)(.15)+(.16)(.10)+(.20)(.15)+(.13)(.10)+(.14)(.10)} = 0.1116$.
3b. Let B now be the event that the M&M is blue, and W denotes the probability it is whole. Then $P(B|W) = \frac{P(B\cap W)}{P(W)} = \frac{(.24)(.90)}{(.24)(.90)+(.13)(.85)+(.16)(.90)+(.20)(.85)+(.13)(.90)+(.14)(.90)} = 0.2445$.

4. The probability that Bob takes exactly n flips is $(1/2)^{n-1}(1/2) = (1/2)^n$, and similarly, the probability that Alice gets no heads at all during n flips of her coin is $(1/2)^n$. Let B_n denote the event that Bob takes exactly n flips to get heads the first time. Let A_n be the probability that Alice gets no heads during n flips of her coin. So the desired probability is $\sum_{n=1}^{\infty} P(B_n \cap A_n) = \sum_{n=1}^{\infty}(1/4)^n = \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = 1/3$.