STAT/MA 41600
In-Class Problem Set #2: August 26, 2016
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Problem Set 2 Answers

1a. The number of outcomes altogether is $6^3 = 216$. The number of outcomes with distinct values is $(6)(5)(4)$. The outcomes are equally likely, so the probability is $(6)(5)(4)/6^3 = 5/9$.
1b. There are $120/6 = 20$ outcomes with $R < G < B$, so the probability is $20/216 = 5/54$.
   As an alternative view, exactly $1/6$ of the outcomes with distinct values have $R < G < B$, so the probability is $(5/9)(1/6) = 5/54$.
1c. Same answer as **1b**, namely, $5/54$.
1d. There are 15 outcomes altogether with $R = G < B$, so the probability is $15/216 = 5/72$.

2a. There are $(52)(51)(50)(49)(48)$ equally likely outcomes, and in $(4)(51)(50)(49)(48)$ of these, the leftmost card is a Jack, so we get $(4)(51)(50)(49)(48)/((52)(51)(50)(49)(48)) = 4/52 = 1/13$.
   Alternatively, we could just focus on the leftmost card. There are 52 equally likely possible values for the leftmost card, and 4 of these are Jacks, so the probability is $4/52 = 1/13$.
2b. Same method, same answer, as in **2a**.
2c. There are $(52)(51)(50)(49)(48) - (48)(47)(46)(49)(48)$ outcomes with at least one Jack among the 2nd, 3rd, 4th cards, so the probability is:
   $((52)(51)(50)(49)(48) - (48)(47)(46)(49)(48))/((52)(51)(50)(49)(48)) = 1201/5525$.
   Alternatively, focusing on the 2nd, 3rd, and 4th cards, the probability is the complement of $(48/52)(47/51)(46/50)$, i.e., it is $1 - (48/52)(47/51)(46/50) = 1201/5525$.
2d. Yes, these answers agree.

3a. The probability of a tail on the last flip is $1/2$.
3b. The number of outcomes altogether is $2^5 = 32$, and $2^4 = 16$ of these outcomes have a head on the 5th flip. The outcomes are equally likely, so the probability is $16/32 = 1/2$. This agrees with the answer from **3a**.
3c. At least 2 heads appear at the start if and only if the first two flips are heads. So we need the 1st and 2nd flips to be heads, and the 5th flip to be tails, so the probability is $(1/2)^3 = 1/8$.

4a. Just like in **1a**, the probability is $5/9$. If you are concerned about the dice having different colors, then just roll them consecutively, or have three different people roll them, or roll them in three separate places on the table, etc.

4b. This event can be split into three non-overlapping parts, i.e., either $R = G < B$ or $R = B < G$ or $B = G < R$, each of which has probability $5/72$, so the desired probability is $5/72 + 5/72 + 5/72 = 15/72 = 5/24$.