STAT/MA 41600
In-Class Problem Set #1: August 24, 2016
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Problem Set 1 Answers

1a. The number of outcomes altogether is $6^3 = 216$.
1b. The number of outcomes with distinct values is (6)(5)(4).
1c. The 120 outcomes in part **1b** can be classified according to whether $R < G < B$ or $R < B < G$ or $B < G < R$ or $B < R < G$ or $G < R < B$ or $G < B < R$, and each of these possibilities has the same number of outcomes, and there are no overlaps, so there are $120/6 = 20$ outcomes with $R < G < B$.

As an alternative view, there are $\binom{6}{3} = \frac{6!}{3!3!} = 20$ ways to pick three different values, and then we let the smallest one be R, the middle one be G, and the largest one be B.
1d. Same answer as **1c**, namely, 20.
1e. There are 5 outcomes with $R = G < B = 6$, and 4 outcomes with $R = G < B = 5$, and 3 outcomes with $R = G < B = 4$, and 2 outcomes with $R = G < B = 3$, and 1 outcome with $R = G < B = 2$. So there are 15 outcomes altogether with $R = G < B$.

As an alternative view, there are $\binom{6}{2} = \frac{6!}{4!2!} = 15$ ways to pick two different values, and then we let the smaller one be R and G, and the larger one be B.

2a. There are (52)(51)(50)(49)(48) outcomes altogether.
2b. In exactly (4)(51)(50)(49)(48) of the outcomes, the leftmost card is a Jack.
2c. In exactly (4)(51)(50)(49)(48) of the outcomes, the rightmost card is a Jack!
2d. There are (48)(47)(46)(49)(48) outcomes in which the 2nd, 3rd, and 4th cards are not Jacks. Therefore, subtracting from **2a**, there are (52)(51)(50)(49)(48) − (48)(47)(46)(49)(48) outcomes in which there is at least one Jack among the 2nd, 3rd, 4th cards.

3a. The number of outcomes altogether is $2^5 = 32$.
3b. Since there are 32 outcomes, there are 2^{32} possible events.
3c. There are $2^4 = 16$ outcomes in which a head occurs on the 5th flip.
3d. There are 2^{31} events that contain the outcome (H, H, T, T, T). To describe any such event, we simply make 31 decisions, namely, whether to include (or not include) each of the other 31 outcomes in the event we are describing.

4a. We have $\int_0^\infty \lambda e^{-\lambda x}\,dx = -e^{-\lambda x}|_{x=0}^\infty = 1$.
4b. We have $\int_a^\infty \lambda e^{-\lambda x}\,dx = -e^{-\lambda x}|_{x=a}^\infty = e^{-\lambda a}$.
4c. We have $\int_0^\infty (x)(\lambda e^{-\lambda x})\,dx = (x)(-e^{-\lambda x})|_{x=0}^\infty - \int_0^\infty (1)(-e^{-\lambda x})\,dx = \int_0^\infty e^{-\lambda x}\,dx = \frac{e^{-\lambda x}}{-\lambda}|_{x=0}^\infty = 1/\lambda$.
4d. We have $\sum_{x=0}^\infty \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda}\sum_{x=0}^\infty \frac{\lambda^x}{x!} = e^{-\lambda}e^{\lambda} = 1$.
4e. We have $\sum_{x=3}^\infty \frac{\lambda^x e^{-\lambda}}{x!} = 1 - \sum_{x=0}^2 \frac{\lambda^x e^{-\lambda}}{x!} = 1 - e^{-\lambda}(1 + \lambda + \lambda^2/2)$.