STAT/MA 41600
In-Class Problem Set #1: August 24, 2016

1. Consider a collection of 3 dice. One die is red, one die is green, and one die is blue. Each die is 6-sided and is numbered from 1 to 6.
1a. If we roll all 3 dice (1 time each), and we write down the results, keeping track of the 3 values that appear, as well as the corresponding colors, how many possible outcomes are in the sample space S? (An outcome is an ordered 3-tuple, e.g., it tells us the values of R for the red die, G for the green die, and B for the blue die.)
1b. How many outcomes have *three distinct values* for R, G, and B, i.e., all different values?
1c. How many outcomes have $R < G < B$?
1d. How many outcomes have $G < R < B$?
1e. How many outcomes have $R = G < B$?

2. Consider a standard deck of 52 cards. Deal 5 cards in a row, left to right, on the table. So an outcome is an ordered 5-tuple of cards.
2a. How many outcomes are there altogether?
2b. In how many of the outcomes is the leftmost card a Jack?
2c. In how many of the outcomes is the rightmost card a Jack?
2d. In how many of the outcomes is there at least one Jack, among the middle three cards on the table, i.e., among the 2nd, 3rd, and 4th cards?

3. Consider 5 consecutive tosses of a coin.
3a. How many outcomes are there?
3b. How many events are there?
3c. How many outcomes have a head on the last flip (the other four flips can be arbitrary)?
3d. How many events contain the outcome (H, H, T, T, T)?

4. Calculus review. Suppose $\lambda > 0$. Find a closed form expression for each of these:
4a. $\int_0^\infty \lambda e^{-\lambda x}\, dx$
4b. $\int_a^\infty \lambda e^{-\lambda x}\, dx$ (assume $a > 0$ is fixed)
4c. $\int_0^\infty (x)(\lambda e^{-\lambda x})\, dx$
4d. $\sum_{x=0}^\infty \frac{\lambda^x e^{-\lambda}}{x!}$
4e. $\sum_{x=3}^\infty \frac{\lambda^x e^{-\lambda}}{x!}$