STAT/MA 41600
In-Class Problem Set #42: December 4, 2015

1. Consider three independent continuous Uniform random variables, each of which has a constant density on $[0, 10]$.

a. Find the density $f_{X_{(1)}}(x_1)$ of the 1st order statistic (i.e., find the density of the min).

b. Find the density $f_{X_{(2)}}(x_2)$ of the 2nd order statistic.

c. Find the density $f_{X_{(3)}}(x_3)$ of the 3rd order statistic (i.e., find the density of the max).

2. Same setup as question #1.

a. Find $\mathbb{E}(X_{(1)})$.

b. Find $\mathbb{E}(X_{(2)})$.

c. Find $\mathbb{E}(X_{(3)})$.

d. Since the sum of the three random variables and the sum of the three order statistics must be the same (always), then their expected values are the same, i.e., $X_1 + X_2 + X_3 = X_{(1)} + X_{(2)} + X_{(3)}$. So $\mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_{(1)} + X_{(2)} + X_{(3)})$. We also know that $\mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 5 + 5 + 5 = 15$. Use this to double check your answers to parts a, b, c. Do the answers sum up to 15?

3. Suppose X_1, X_2 are independent continuous random variables with $f_{X_1,X_2}(x_1, x_2) = (1/8)^2(4 - x_1)(4 - x_2)$ on the square $0 < x_1 < 4$ and $0 < x_2 < 4$, and $f_{X_1,X_2}(x_1, x_2) = 0$ otherwise.

a. Find the density $f_{X_{(1)}}(x_1)$ of the 1st order statistic (i.e., find the density of the min).

b. Find the density $f_{X_{(2)}}(x_2)$ of the 2nd order statistic (i.e., find the density of the max).

4. Same setup as question #3.

a. Find $\mathbb{E}(X_{(1)})$.

b. Find $\mathbb{E}(X_{(2)})$.

c. Since the sum of the two random variables and the sum of the two order statistics must be the same (always), then their expected values are the same, i.e., $X_1 + X_2 = X_{(1)} + X_{(2)}$. So $\mathbb{E}(X_1 + X_2) = \mathbb{E}(X_{(1)} + X_{(2)})$. We also know that $\mathbb{E}(X_1 + X_2) = \mathbb{E}(X_1) + \mathbb{E}(X_2) = 4/3 + 4/3 = 8/3$. Use this to double check your answers to parts a, b. Do the answers sum up to $8/3$?