STAT/MA 41600

In-Class Problem Set #41: December 2, 2015 Solutions by Mark Daniel Ward

1a. If X denotes the grade, then $P(X \ge 0.95) \le \frac{\mathbb{E}(X)}{0.95} = \frac{0.80}{0.95} = 0.84$. **1b.** We have $P(0.73 \le X \le 0.87) = P(|X - 0.80| \le 0.07) = P(|X - 0.80| \le k\sigma_X)$ where $\sigma_X = 0.05$ is the standard deviation, and k = 0.07/0.05. So we have $P(0.73 \le X \le 0.87) \ge$ $1 - \frac{1}{(0.07/0.05)^2} = 0.49.$

2a. We have $P(|X - 22| \ge 0.5) = P(|X - 22| \ge k\sigma_X)$ where $\sigma_X = 0.3$ and k = 0.5/0.3. So we get $P(|X - 22| \ge 0.5) \le \frac{1}{(0.5/0.3)^2} = 0.36.$

2b. We have $P(X \ge 24) \le \frac{22}{24} = 0.92$, by the Markov inequality.

2c. We have $P(X \ge 21) \le \frac{22}{21} = 1.05$, but of course we automatically have an even better bound (without using the Markov inequality), namely, $P(X \ge 21) \le 1$. So the Markov inequality does not give us any additional information in this case.

3a. We use X for the number of bees. Then we get $P(X \ge 20) \le \frac{15}{20} = 0.75$.

3b. We have $P(10 \le X \le 20) = P(|X - 10| \le 5) = P(|X - 10| \le k\sigma_X)$ where $\sigma_X = 3$ and k = 5/3. So we get $P(10 \le X \le 20) \ge 1 - \frac{1}{(5/3)^2} = 0.64$.

4a. We use X to denote the number of customers. Then $P(20 \le X \le 40) = P(|X - X|)$ $|30| \le 10$ = $P(|X - 30| \le k\sigma_X)$ where $\sigma_X = 5$ and k = 10/5 = 2. So we conclude $P(20 \le X \le 40) \ge 1 - \frac{1}{2^2} = 3/4.$

4b. We have $P(X \ge 40) \le \frac{30}{40} = 3/4 = 0.75$.

4c. We have $P(X \ge 50) \le \frac{30}{50} = 3/5 = 0.60$.

4d. We have $P(X \ge 60) \le \frac{30}{60} = 1/2 = 0.50$.