STAT/MA 41600
In-Class Problem Set #40: November 30, 2015

1. Suppose X and Y have joint probability density function

$$f_{X,Y}(x, y) = 70e^{-3x-7y}$$

for $0 < x < y$; and $f_{X,Y}(x, y) = 0$ otherwise.
  1a. Find the probability density function $f_X(x)$ of X.
  1b. Use your solution to **1a** to find $f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$ for fixed $x > 0$.
  1c. Use your solution to **1b** to find $\mathbb{E}(Y \mid X = x) = \int_x^\infty y f_{Y|X}(y \mid x)\, dy$, for a fixed $x > 0$.
  1d. Use your solution to **1c** to find $\mathbb{E}(Y) = \int_0^\infty \mathbb{E}(Y \mid X = x) f_X(x)\, dx$.
  [Hint: The new thing here is question 1c. We solved 1a and 1b in Problem Set 27. Also, you can compare your answer to 1d with the answer from question 2 in Problem Set 28.]

2. Roll two 4-sided dice. Let X denote the maximum value, and let Y denote the minimum value.
  2a. Find $\mathbb{E}(X \mid Y = 3)$.
  2b. Find $\mathbb{E}(Y \mid X = 3)$.

3. Consider a pair of continuous random variables X, Y with constant joint density on the triangle with vertices at $(0, 0)$, $(2, 0)$, and $(0, 8)$.
  3a. Fix y with $0 < y < 8$. Find $\mathbb{E}(X \mid Y = y)$.
  3b. Fix x with $0 < x < 2$. Find $\mathbb{E}(Y \mid X = x)$.

4. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets, and let X denote the number of red marbles that Bob gets.
  4a. Find $\mathbb{E}(X \mid Y = 0)$.
  4b. Find $\mathbb{E}(X \mid Y = 1)$.
  4c. Find $\mathbb{E}(X \mid Y = 2)$.