STAT/MA 41600
In-Class Problem Set #39 part 2: November 23, 2015

1. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. Roll both dice, and let X denote the number of red sides that appear. Find the variance of X.

2. A bag of candy contains 10 green M&M's and 10 red M&M's. Suppose that 10 students pick 2 candies each, without replacement. Let X denote the number of students who get one red and one green candy. Find $\mathrm{Var}(X)$.

3. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0,0)$, $(5,0)$, and $(0,5)$. Find the covariance of X and Y.

4. Suppose X and Y have joint density $f_{X,Y}(x,y) = e^{1-x}$ for x, y in the region where $0 < x < y < 1$, and $f_{X,Y}(x,y) = 0$ otherwise. Find the covariance of X and Y.

[Note: It might look strange to have a joint probability density function of X and Y with no y's in it, but this is OK. This function is constant with regard to y, i.e., it does not change as y changes. You can check, for instance, that $f_{X,Y}(x,y)$ is a valid probability density function because it is nonnegative and because $\int_0^1 \int_x^1 e^{1-x}\,dy\,dx = 1$.]

[Hint: Just to save you having to do so many integration by parts, for your convenience, we have: $\int_0^1 e^{-x}\,dx = 1 - e^{-1}$ and $\int_0^1 xe^{-x}\,dx = 1 - 2e^{-1}$ and $\int_0^1 x^2e^{-x}\,dx = 2 - 5e^{-1}$ and $\int_0^1 x^3e^{-x}\,dx = 6 - 16e^{-1}$.]