STAT/MA 41600
In-Class Problem Set #39: November 20, 2015

1. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). Let X denote the number of red bears that are chosen, among these 5 selected bears. Find the variance of X.

2. Consider 5 fish in a bowl: 3 of them are red, and 1 is green, and 1 is blue. Select the fish one at a time, without replacement, until the bowl is empty.

Let $X = 1$ if all of the red fish are selected, before the green fish is selected; and $X = 0$ otherwise.

Let $Y = 1$ if all of the red fish are selected, before the blue fish is selected; and $Y = 0$ otherwise.

2a. Find the covariance of X and Y.

2b. Find the correlation of X and Y.

3. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets, and let X denote the number of red marbles that Bob gets.

3a. Find the covariance of X and Y.

3b. Find the correlation of X and Y.

4. Suppose X and Y have joint density $f_{X,Y}(x, y) = 10e^{-3x-2y}$ for x, y in the region where $0 < x < y$, and $f_{X,Y}(x, y) = 0$ otherwise.

4a. (Review) Find the density $f_Y(y)$ of Y.

4b. Find the covariance of X and Y.

Hint: As seen in Problem Set #26, question 2b, we have $f_X(x) = \int_x^\infty 10e^{-3x-2y}\,dy = 5e^{-5x}$ for $x > 0$, and $f_X(x) = 0$ otherwise.