STAT/MA 41600
In-Class Problem Set #36: November 11, 2015
Solutions by Mark Daniel Ward

1a. Use $X_1, \ldots, X_{40}$ to denote the 40 liquid amounts, so $P(7.9 \le X_1 + \cdots + X_{40} \le 8.1) = P\big(\frac{7.9-40(0.20)}{\sqrt{40(0.05)^2}} \le \frac{X_1+\cdots+X_{40}-40(0.20)}{\sqrt{40(0.05)^2}} \le \frac{8.1-40(0.20)}{\sqrt{40(0.05)^2}}\big) = P(-0.32 \le Z \le 0.32) = 0.6255 - (1 - 0.6255) = 0.2510.$

1b. We have $0.95 = P(8-b \le X_1+\cdots+X_{40} \le 8+b) = P\big(\frac{8-b-40(0.20)}{\sqrt{40(0.05)^2}} \le \frac{X_1+\cdots+X_{40}-40(0.20)}{\sqrt{40(0.05)^2}} \le \frac{8+b-40(0.20)}{\sqrt{40(0.05)^2}}\big) = P\big(-\frac{b}{0.32} \le Z \le \frac{b}{0.32}\big) = P\big(Z \le \frac{b}{0.32}\big) - \big(1 - P\big(Z \le \frac{b}{0.32}\big)\big) = 2P\big(Z \le \frac{b}{0.32}\big) - 1.$ So $P\big(Z \le \frac{b}{0.32}\big) = 0.975$. Thus $\frac{b}{0.32} = 1.96$. So we get $b = (0.32)(1.96) = 0.6272$.

2a. Use $X_1, \ldots, X_{5000}$ to denote the weights of the stones, so $P(349000 \le X_1+\cdots+X_{5000}) = P\big(\frac{349000-5000(70)}{\sqrt{5000(8)^2}} \le \frac{X_1+\cdots+X_{40}-5000(70)}{\sqrt{5000(8)^2}}\big) = P(-1.77 \le Z) = P(1.77 \ge Z) = 0.9616.$

2b. We have $P(\mu-500 \le X_1+\cdots+X_{5000} \le \mu+500) = P\big(\frac{\mu-500-5000(70)}{\sqrt{5000(8)^2}} \le \frac{X_1+\cdots+X_{5000}-5000(70)}{\sqrt{5000(8)^2}} \le \frac{\mu+500-5000(70)}{\sqrt{5000(8)^2}}\big) = P\big(-\frac{500}{565.69} \le Z \le \frac{500}{565.69}\big) = P(-0.88 \le Z \le 0.88) = P(Z \le 0.88) - (1 - P(Z \le 0.88)) = 0.8106 - (1 - 0.8106) = 0.6212.$

3a. We have $P\big(3.5 \le \frac{X_1+\cdots+X_{10}}{10} \le 4.5\big) = P((3.5)(10) \le X_1 + \cdots + X_{10} \le (4.5)(10)) = P\big(\frac{(3.5)(10)-10(4)}{\sqrt{10(0.75)^2}} \le \frac{X_1+\cdots+X_{10}-10(4)}{\sqrt{10(0.75)^2}} \le \frac{(4.5)(10)-10(4)}{\sqrt{10(0.75)^2}}\big) = P(-2.11 \le Z \le 2.11) = 0.9826 - (1 - 0.9826) = 0.9652.$

3b. We have $0.90 = P\big(4-a \le \frac{X_1+\cdots+X_{10}}{10} \le 4+a\big) = P((4-a)(10) \le X_1 + \cdots + X_{10} \le (4+a)(10)) = P\big(\frac{(4-a)(10)-10(4)}{\sqrt{10(0.75)^2}} \le \frac{X_1+\cdots+X_{10}-10(4)}{\sqrt{10(0.75)^2}} \le \frac{(4+a)(10)-10(4)}{\sqrt{10(0.75)^2}}\big) = P\big(-\frac{10a}{2.37} \le Z \le \frac{10a}{2.37}\big) = P\big(Z \le \frac{10a}{2.37}\big) - \big(1 - P\big(Z \le \frac{10a}{2.37}\big)\big) = 2P\big(Z \le \frac{10a}{2.37}\big) - 1.$ So $P\big(Z \le \frac{10a}{2.37}\big) = 0.95$. Thus $\frac{10a}{2.37} = 1.645$. So we get $a = (2.37)(1.645)/10 = 0.3899$.

4a. Use $X_1, \ldots, X_5$ to denote the weights of the encyclopedias and $Y_1, \ldots, Y_{20}$ to denote the weights of the novels, so we compute $P(X_1 + \cdots + X_5 + Y_1 + \cdots + Y_{20} \le 60) = P\big(\frac{X_1+\cdots+X_5+Y_1+\cdots+Y_{20}-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \le \frac{60-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}}\big) = P(Z \le 0.89) = 0.8133.$

4b. We compute $P(58 \le X_1 + \cdots + X_5 + Y_1 + \cdots + Y_{20} \le 62) = P\big(\frac{58-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \le \frac{X_1+\cdots+X_5+Y_1+\cdots+Y_{20}-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \le \frac{62-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}}\big) = P(0 \le Z \le 1.79) = 0.9633 - 0.5000 = 0.4633.$