STAT/MA 41600
In-Class Problem Set #35: November 9, 2015
Solutions by Mark Daniel Ward

1a. Using Z to denote a standard normal random variable, we have $P(1 < X < 2) = P(\frac{1-1.2}{0.5} < \frac{X-1.2}{0.5} < \frac{2-1.2}{0.5}) = P(-0.4 < Z < 1.6) = P(Z < 1.6) - P(Z \leq -0.4)$. Checking our table, we have $P(Z < 1.6) = 0.9452$ and $P(Z \leq -0.4) = P(Z \geq 0.4) = 1 - P(Z < 0.4) = 1 - 0.6554 = 0.3446$. Thus $P(1 < X < 2) = 0.9452 - 0.3446 = 0.6006$.

1b. We have $P(X > 1.4) = P(\frac{X-1.2}{0.5} > \frac{1.4-1.2}{0.5}) = P(Z > 0.4) = 1 - P(Z \leq 0.4) = 1 - 0.6554 = 0.3446$. Also, we have $P(X < 1) = P(\frac{X-1.2}{0.5} < \frac{1-1.2}{0.5}) = P(Z < -0.4) = P(Z > 0.4) = 1 - P(Z \leq 0.4) = 1 - 0.6554 = 0.3446$. So the desired probability is $P(X > 1.4 \text{ or } X < 1) = 0.3446 + 0.3446 = 0.6892$.

1c. We compute $P(X \geq 0) = P(\frac{X-1.2}{0.5} \geq \frac{0-1.2}{0.5}) = P(Z \geq -2.40) = P(Z \leq 2.40) = 0.9918$.

2a. We compute $0.10 = P(X \leq a) = P(\frac{X-1.2}{0.5} \leq \frac{a-1.2}{0.5}) = P(Z \leq \frac{a-1.2}{0.5})$. Thus $0.90 = 1 - 0.10 = 1 - P(Z \leq \frac{a-1.2}{0.5}) = P(Z > \frac{a-1.2}{0.5}) = P(Z < -\frac{a-1.2}{0.5})$. So we must have $-\frac{a-1.2}{0.5} = 1.28$. It follows that $a = (-1.28)(0.5) + 1.2 = 0.56$.

2b. We compute $0.10 = P(X \geq b) = P(\frac{X-1.2}{0.5} \geq \frac{b-1.2}{0.5}) = P(Z \geq \frac{b-1.2}{0.5})$. Thus $0.90 = 1 - 0.10 = 1 - P(Z \geq \frac{b-1.2}{0.5}) = P(Z < \frac{b-1.2}{0.5})$. So we must have $\frac{b-1.2}{0.5} = 1.28$. It follows that $b = (1.28)(0.5) + 1.2 = 1.84$.

2c. We compute that: $0.30 = P(1.2 - c < X < 1.2 + c) = P(\frac{-c}{0.5} < \frac{X-1.2}{0.5} < \frac{c}{0.5}) = P(\frac{-c}{0.5} < Z < \frac{c}{0.5}) = P(Z < \frac{c}{0.5}) - P(Z \leq \frac{-c}{0.5})$. The second term of this last part is $P(Z \leq \frac{-c}{0.5}) = P(Z \geq \frac{c}{0.5}) = 1 - P(Z < \frac{c}{0.5})$. So altogether we get $0.30 = 2P(Z < \frac{c}{0.5}) - 1$. So $1.30 = 2P(Z < \frac{c}{0.5})$ and $0.65 = P(Z < \frac{c}{0.5})$. Thus $\frac{c}{0.5} = 0.385$. So $c = (0.385)(0.5) = 0.193$.

3. Let X denote a student's score.

The probability of an A grade is $P(90 < X < 100) = P(\frac{90-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{100-72.5}{6.9}) = P(2.54 < Z < 3.98) = P(Z < 3.98) - P(Z \leq 2.54) = 1.0000 - 0.9945 = 0.0055$.

The probability of a B grade is $P(80 < X < 90) = P(\frac{80-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{90-72.5}{6.9}) = P(1.09 < Z < 2.54) = P(Z < 2.54) - P(Z \leq 1.09) = 0.9945 - 0.8621 = 0.1324$.

The probability of a C grade is $P(70 < X < 80) = P(\frac{70-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{80-72.5}{6.9}) = P(-0.36 < Z < 1.09) = P(Z < 1.09) - P(Z \leq -0.36)$. We have $P(Z < 1.09) = 0.8621$ and $P(Z \leq -0.36) = P(Z \geq 0.36) = 1 - P(Z < 0.36) = 1 - 0.6406 = 0.3594$. So $P(70 < X < 80) = 0.8621 - 0.3594 = 0.5027$.

The probability of a D grade is $P(60 < X < 70) = P(\frac{60-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{70-72.5}{6.9}) = P(-1.81 < Z < -0.36) = P(Z < -0.36) - P(Z \leq -1.81)$. We have $P(Z < -0.36) = 0.3594$ (as in the previous part) and $P(Z \leq -1.81) = P(Z \geq 1.81) = 1 - P(Z < 1.81) = 1 - 0.9649 = 0.0351$. So $P(60 < X < 70) = 0.3594 - 0.0351 = 0.3243$.

4a. If X is the length of the blade of grass in inches, we have $P(X \leq \frac{9}{2.54}) = P(X \leq 3.54) = P(\frac{X-4}{0.75} \leq \frac{3.54-4}{0.75}) = P(Z \leq -0.61) = P(Z \geq 0.61) = 1 - P(Z < 0.61) = 1 - 0.7291 = 0.2709$.

4b. We have $0.90 = P(4 - a < X < 4 + a) = P(\frac{-a}{0.75} < \frac{X-4}{0.75} < \frac{a}{0.75}) = P(\frac{-a}{0.75} < Z < \frac{a}{0.75}) = P(Z < \frac{a}{0.75}) - P(Z \leq \frac{-a}{0.75})$. The second term is $P(Z \leq \frac{-a}{0.75}) = P(Z \geq \frac{a}{0.75}) = 1 - P(Z < \frac{a}{0.75})$. Thus $0.90 = 2P(Z < \frac{a}{0.75}) - 1$. So $1.90 = 2P(Z < \frac{a}{0.75})$, and thus $0.95 = P(Z < \frac{a}{0.75})$. So we get $\frac{a}{0.75} = 1.65$. So the desired a is $(0.75)(1.65) = 1.24$.