STAT/MA 41600
In-Class Problem Set #33: November 4, 2015

1. We know the form of the CDF of a Gamma random variable X with general r, λ is $F_X(x) = 1 - e^{-\lambda x} \sum_{j=0}^{r-1} \frac{(\lambda x)^j}{j!}$ for $x > 0$, and $F_X(x) = 0$ otherwise, but we didn't prove/show it.

1a. Setup and then calculate the double integral for the CDF of a Gamma random variable, in the specific case where $r = 2$ and where $\lambda > 0$ is a fixed positive constant.

[Hint: we did this in 1b on Monday already, in the case where $\lambda = 1/5$. So you can just reuse your work from 1b on Monday with $1/5$ replaced by λ. On the other hand, there is some merit in setting the double integral up and solving it, without peeking at your work from Monday first, as a self-test that you remember how to do 1b from Monday.]

1b. Setup—but you do not need to calculate!—the triple integral that would give you the CDF for a Gamma random variable with $r = 3$ and λ.

2a. Suppose that U and V have joint probability density function $f_{U,V}(u, v) = 9e^{-3u-3v}$ for u and v positive, and $f_{U,V}(u, v) = 0$ otherwise.

2a. Are U and V independent?

2b. What is the density of U?

2c. If we define $X = U + V$, what kind of random variable is X? What are its parameters? What is the density of X?

2d. Can you find $P(X \leq 1/2)$? You can either use the general formula given in the setup of question 1, or you can use the result of your answer to 1a, or you can setup the integral using the density from 2c and calculate directly.

3. Same setup as #2.

3a. What is $P(U > V)$? Hint: You shouldn't have to calculate anything to solve this.

3b. Let $Y = 0$ if $U > 1/10$ and $V > 1/10$.
Let $Y = 2$ if $U \leq 1/10$ and $V \leq 1/10$.
Let $Y = 1$ if $U > 1/10$ and $V \leq 1/10$, or if $U \leq 1/10$ and $V > 1/10$.

In other words, let Y count how many of the variables U and/or V are less than $1/10$. What kind of random variable is Y? What are the parameters of Y?

4. Suppose that Alfredo, Bruno, and Charlie wait for their (respective) girlfriends outside of their classes, and their girlfriends are all in different classes, so their waiting times are independent. Let X, Y, and Z denote their (respective) waiting times, given in minutes. Suppose these waiting times are each Exponential random variables, with average of 5 minutes each.

Now define $W = X + Y + Z$, i.e., their total waiting time, given in minutes.

4a. What kind of random variable is W? What is the density of W?

4b. What is $\text{Var}(W)$? (You do not need to calculate any integrals. You can simply find this by the general formula for the variance of the sum of independent random variables, and you should check that your answer agrees with what you know about the variance of the type of random variable that W is.)

4c. Notice that $W/60$ is their total waiting time, given in hours. What is $\text{Var}(W/60)$?