STAT/MA 41600
In-Class Problem Set #31: October 28, 2015
(there is no Problem Set #30)

1. Consider a pair of random variables X, Y with constant joint density on the quadrilateral with vertices located at the points $(0,0)$, $(3,0)$, $(5,2)$, $(0,2)$.

1a. Find $P(X > Y)$.

1b. Find $P(X + Y \leq 3)$.

2. Suppose X and Y have a constant joint density on the square with vertices $(0,0)$, $(0,5)$, $(5,5)$, $(5,0)$. Find $\mathbb{E}(\max(X,Y))$.

3. Suppose that I am standing at the exact center of a rectangular region with corners $(0,0)$, $(6,0)$, $(6,4)$, $(0,4)$, i.e., I am standing exactly at the point $(3,2)$. A butterfly lands somewhere on the ground, uniformly at random within this rectangular region. What is the probability that the butterfly lands within 1 unit of me, i.e., what is the probability that we are less than 1 unit away from each other?

4. Suppose X has density $f_X(x) = 1/2$ for $0 < x < 2$, and $f_X(x) = 0$ otherwise. Suppose that Y has density $f_Y(y) = e^{-y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Also suppose that X and Y are independent. Find $P(Y > X)$.