STAT/MA 41600

In-Class Problem Set #26: October 19, 2015

1. Consider a pair of random variables X and Y with joint probability density function $f_{X,Y}(x,y) = \frac{1}{8}xy$ for x, y in the triangle where $0 < x < 2$ and $0 < y < 2x$, and $f_{X,Y}(x,y) = 0$ otherwise.

1a. Are X and Y independent? Why or why not?

1b. Find $P(X \leq 1)$ using the joint density $f_{X,Y}(x,y)$.

1c. Find the density $f_X(x)$.

1d. Use the density $f_X(x)$ to find $P(X \leq 1)$. Does your answer agree with your answer to **b**?

2. Suppose X and Y have joint density $f_{X,Y}(x,y) = 10e^{-3x-2y}$ for x, y in the region where $0 < x < y$, and $f_{X,Y}(x,y) = 0$ otherwise.

2a. Find $P(Y > 2X)$. (Just a side comment, not a hint: We already know $P(Y > X) = 1$.)

2b. Find the density $f_X(x)$ of X.

3. Suppose X, Y has joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{225}(5-x)(6-y) & \text{if } 0 \leq x \leq 5 \text{ and } 0 \leq y \leq 6, \\ 0 & \text{otherwise.} \end{cases}$$

3a. Are X and Y independent? Why or why not?

3b. Find the density $f_X(x)$ of X.

3c. Find the density $f_Y(y)$ of Y.

4. Suppose X is a continuous random variable with density $f_X(x) = 3e^{-3x}$ for $x > 0$, and $f_X(x) = 0$ otherwise. Suppose Y is a continuous random variable with density $f_Y(y) = 5e^{-5y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Finally, suppose that X and Y are independent. Define Z as the minimum of X and Y, i.e., $Z = \min(X, Y)$.

4a. Find the density $f_Z(z)$ of Z.

4b. Find $P(Z > 1/10)$.