STAT/MA 41600
In-Class Problem Set #20/#22: October 5, 2015

1. A playlist contains 10 rock songs, 3 country songs, 5 R&B songs, and 2 blues songs. In shuffle mode, each song is played exactly once, and all possible equal orderings are equally likely. Suppose that a person starts this playlist in shuffle mode and continues until a country music song plays, and then stops. Let X denote the number of songs played *before* the country music song (but not including the country music song itself). [[Hint: Write $X = X_1 + \cdots + X_{17}$, where $X_j = 1$ if the jth non-country song is played before all of the country songs, or $X_j = 0$ otherwise.]]

1a. Find $\mathbb{E}(X)$.

1b. Find $\mathrm{Var}(X)$.

2. In question (1a), suppose that we randomly pick 4 songs (without repetitions) to play.

2a. What is the probability that we get 1 song from each of the 4 genres?

2b. What is the probability that all 4 songs are selected from the same 1 genre?

2c. What is the probability that 3 of the 4 genres appear during the 4 songs?

2d. Knowing the answers to a, b, c, you can use the complement to find the probability that 2 of the 4 genres appear during the 4 songs. To test your strength, are you able to also calculate this probability directly?

3. A bag of candy contains 10 green M&M's and 10 red M&M's. Suppose that 10 students pick 2 candies each, without replacement. Let X denote the number of students who get one red and one green candy.

3a. Find $\mathbb{E}(X)$.

3b. Find $\mathrm{Var}(X)$.

4. Consider the even positive integers $2, 4, 6, \ldots, 100$. Let X be one of these integers, with all selections equally likely.

4a. Find $\mathbb{E}(X)$.

4b. Find $\mathrm{Var}(X)$.

[[Hint: You might find it helpful to use the fact that $X = 2Y$, where Y is a Discrete Uniform random variable.]]