STAT/MA 41600
In-Class Problem Set #19: October 2, 2015
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Problem Set 19 Answers

1a. Since X is Binomial with $n = 4$ and $p = 7/20$, then $\mathbb{E}(X) = np = (4)(7/20) = 7/5$, and $\text{Var}\,(X) = np(1-p) = (4)(7/20)(13/20) = 91/100$.
1b. Since Y is Hypergeometric with $N = 20$, $M = 7$, and $n = 4$, then we get $\mathbb{E}(Y) = n(M/N) = (4)(7/20) = 7/5$, and $\text{Var}\,(Y) = n(M/N)(1 - M/N)(N-n)/(N-1) = (4)(7/20)(1-7/20)(20-4)/(20-1) = 364/475 = 0.7663$.

2. We have $P(X = Y) = P(X = Y = 0) + P(X = Y = 1) + P(X = Y = 2) = (1/5)^2 + (3/5)^2 + (1/5)^2 = 11/25$.

3a. The exact expression is $P(X = 4) = \binom{15000}{4}\binom{35000}{6}/\binom{50000}{10} = 0.20013524\ldots$ (You did not have to put the decimal value, of course; it is probably way too large for your calculator.)
3b. Since X is approximately Binomial with $n = 10$ and $p = M/N = 35000/50000 = 7/10$, then $P(X = 4)$ is approximately equal to $\binom{10}{4}(3/10)^4(7/10)^6 = 0.20012095\ldots$

4a. If $n = 1$, then $P(X = 1) = \binom{1}{1}p^1(1-p)^{1-1} = p$ and $P(Y = 1) = M/N$, so these are the same value. The intuitive reason is that X corresponds to a sampling of one item with replacement, to see if it is a success, and Y corresponds to a sampling of one item without replacement, to see if it a success, but we don't worry about whether or not we are replacing after picking, because we only pick one item to test.
4b. We have $P(X = n) = \binom{n}{n}p^n(1-p)^{n-n} = p^n$, which is equal to $(M/N)^n$. In contrast, $P(Y = n) = (\frac{M}{N})(\frac{M-1}{N-1})(\frac{M-2}{N-2})\cdots(\frac{M-n+1}{N-n+1}) < (M/N)^n$, so $P(Y = n) < P(X = n)$.