STAT/MA 41600
In-Class Problem Set #17: September 28, 2015
Solutions by Mark Daniel Ward

Problem Set 17 Answers

1a. The probability that she solves 8 or fewer questions is

$$\binom{4}{4}(1/10)^0(9/10)^5 + \binom{5}{4}(1/10)^1(9/10)^5 + \binom{6}{4}(1/10)^2(9/10)^5 + \binom{7}{4}(1/10)^3(9/10)^5$$

which simplifies to $0.5905 + 0.2952 + 0.08857 + 0.02067 = 0.995$.
1b. The probability that she solves 6 or fewer questions is

$$\binom{4}{4}(1/10)^0(9/10)^5 + \binom{5}{4}(1/10)^1(9/10)^5 = 0.5905 + 0.2952 = 0.886.$$

So the desired conditional probability is $0.886/0.995 = 0.890$.
1c. The variance is $rq/p^2 = (5)(1/10)/(9/10)^2 = 50/81 = 0.6173$.

2a. We have $P(X > 6) = 1 - P(X \leq 6)$, and

$$\begin{aligned} P(X \leq 6) &= \binom{2}{2}(4/10)^0(6/10)^3 + \binom{3}{2}(4/10)^1(6/10)^3 + \binom{4}{2}(4/10)^2(6/10)^3 + \binom{5}{2}(4/10)^3(6/10)^3 \\ &= 0.216 + 0.2592 + 0.20736 + 0.13824 = 0.8208. \end{aligned}$$

So $P(X > 6) = 1 - 0.8208 = 0.1792$.
2b. We have $P(X > 4) = 1 - P(X \leq 4)$, and

$$P(X \leq 4) = \binom{2}{2}(4/10)^0(6/10)^3 + \binom{3}{2}(4/10)^1(6/10)^3 = 0.216 + 0.2592 = 0.4752.$$

So $P(X > 4) = 1 - 0.4752 = 0.5248$. We conclude that $P(X > 6 \mid X > 4) = \frac{P(X>6 \ \& \ X>4)}{P(X>4)} = \frac{P(X>6)}{P(X>4)} = 0.1792/0.5248 = 0.3415$.

3a. Yes, $X_1+X_2+X_3$ is a Negative Binomial random variable with $r = 3$ and $p = 1/(10/7) = 7/10$. So $X_1 + X_2 + X_3$ has the same distribution as Y.
3b. No, $X_1 + X_2 + X_3$ and Z do not have the same distribution, because $X_1 + X_2 + X_3$ can take on any positive integer values of 3 or larger, but Z can only take on values that are positive integer multiples of 3.
3c. No, Y and Z do not have the same distribution, because Y can take on any positive integer values of 3 or larger, but Z can only take on values that are positive integer multiples of 3.

4a. No, X is not a Negative Binomial random variable. Instead, X is the sum of 6 independent random variables, each of which has a different value of p.
4b. We have $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_6) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_6) = \frac{1}{6/6} + \frac{1}{5/6} + \frac{1}{4/6} + \frac{1}{3/6} + \frac{1}{2/6} + \frac{1}{1/6} = 147/10 = 14.7$.