STAT/MA 41600
In-Class Problem Set #17: September 28, 2015

1. Suppose Jessica picks homework problems at random to practice for her midterm exam. She practices until she has solved 5 worthwhile questions, and then she quits after that. Her selections of problems are independent, each with a probability of 0.90 of being worthwhile.

1a. Find the probability that she solves 8 or fewer questions.

1b. Find the conditional probability that she solves 6 or fewer questions, given that she solves 8 or fewer questions.

1c. Find the variance of the total number of questions that she solves.

2. Suppose that 60% of people in Chicago are fans of da Bears. Assume that the fans' preferences are independent. We interview fans until we find the 3rd person who is a fan of da Bears. (This is different than the setup from question #1 of Problem Set 12. Here we need to interview at least 3 people, but in that former question, we interviewed exactly 3 people.) Let X denote the number of people we interview altogether.

2a. Find the probability that $X > 6$.

2b. Find the conditional probability that $X > 6$, given that $X > 4$.

3. Let X_1, X_2, X_3 be independent Geometric random variables, each with expected value $10/7$. Let Y be a Negative Binomial random variable, with parameters $r = 3$ and $p = 7/10$. Let $Z = 3X_1$.

3a. Do $X_1 + X_2 + X_3$ and Y have the same distribution? Why or why not?

3b. Do $X_1 + X_2 + X_3$ and Z have the same distribution? Why or why not?

3c. Do Y and Z have the same distribution? Why or why not?

4. Roll a 6 sided die until you have seen all of the sides as a result. Let X denote the number of rolls required.

4a. Is X a negative binomial random variable? If so, what are the parameters? If not, then why not?

4b. Find $\mathbb{E}(X)$.

Hint: Let X_i denote the number of additional rolls needed until the ith new result appears. So $X = X_1 + \cdots + X_6$.