STAT/MA 41600
In-Class Problem Set #5: September 4, 2015
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Problem Set 5 Answers

1a. Let S, G, R denote (respectively) the probability that the car is silver, gray, or red. So we have $P(S \mid S \cup G \cup R) = \frac{P(S \cap (S \cup G \cup R))}{P(S \cup G \cup R)}$. In the numerator, we have $S \cap (S \cup G \cup R)) = S$, because a car is only in S and in $S \cup G \cup R$ if it is (indeed) in S! So we get $P(S \mid S \cup G \cup R) = \frac{P(S)}{P(S \cup G \cup R)}$. The events S, G, and R are disjoint, so this yields $P(S \mid S \cup G \cup R) = \frac{P(S)}{P(S)+P(G)+P(R)} = \frac{.16}{.16+.13+.10} = 0.4103$.
1b. Similar to part 1a, we have $P(G \mid S \cup G \cup R) = \frac{P(G)}{P(S)+P(G)+P(R)} = \frac{.13}{.16+.13+.10} = 0.3333$.
1c. Similar to part 1a, we have $P(R \mid S \cup G \cup R) = \frac{P(R)}{P(S)+P(G)+P(R)} = \frac{.10}{.16+.13+.10} = 0.2564$.

2. Let C be the event that it is a country song, and let F be the event that the selected song has a fiddle. So $P(C \mid F) = \frac{P(C \cap F)}{P(F)} = \frac{(.15)(.90)}{(.15)(.90)+(.21)(.18)+(.24)(0)+(.40)(.10)} = 0.6344$.

3. Let B_1, B_2, B_3 denote the events that the 4-sided die has a result of 1, 2, or 3 (resp.). Let A be the event that the sum of the dice is 5 or larger. Then $P(A \mid B_1 \cup B_2 \cup B_3) = \frac{P(A \cap (B_1 \cup B_2 \cup B_3))}{P(B_1 \cup B_2 \cup B_3)} = \frac{P(A \cap B_1)+P(A \cap B_2)+P(A \cap B_3)}{P(B_1)+P(B_2)+P(B_3)}$, where the last equality is true since the B_j's are disjoint. Also $P(A \cap B_j) = P(B_j)P(A \mid B_j)$, so we get $P(A \mid B_1 \cup B_2 \cup B_3) = \frac{P(B_1)P(A \mid B_1)+P(B_2)P(A \mid B_2)+P(B_3)P(A \mid B_3)}{P(B_1)+P(B_2)+P(B_3)} = \frac{(1/4)(3/6)+(1/4)(4/6)+(1/4)(5/6)}{1/4+1/4+1/4} = 2/3$.

An alternative method is to recognize that there are 18 equally likely outcomes in which the 4-sided die has a result of 1, 2, or 3, and exactly 12 of these 18 outcomes has a sum of 5 or larger on the dice, so the desired probability is $12/18 = 2/3$.

4a. We have

$$\begin{aligned} P(A \mid B^c) = \frac{P(A \cap B^c)}{P(B^c)} &= \frac{P(A)P(B^c \mid A)}{P(A)P(B^c \mid A) + P(A^c)P(B^c \mid A^c)} \\ &= \frac{(1/7)(4/5)}{(1/7)(4/5) + (6/7)((2/6)(1) + (4/6)(4/5))} \\ &= 2/15. \end{aligned}$$

4b. We have

$$\begin{aligned} P(A^c \mid B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} &= \frac{P(A^c)P(B^c \mid A^c)}{P(A)P(B^c \mid A) + P(A^c)P(B^c \mid A^c)} \\ &= \frac{(6/7)((2/6)(1) + (4/6)(4/5))}{(1/7)(4/5) + (6/7)((2/6)(1) + (4/6)(4/5))} \\ &= 13/15. \end{aligned}$$