STAT/MA 41600
In-Class Problem Set #4: September 2, 2015
Solutions by Mark Daniel Ward

Problem Set 4 Answers

1a. If A is the event that the first die was used, and B is the event that red appears, then we have $P(A \cap B) = (1/2)(2/6) = 2/12$ and $P(B) = (1/2)(2/6) + (1/2)(3/6) = 5/12$, so $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{2/12}{5/12} = 2/5$.

Another method of solution is to note that there are 5 ways that red can appear, and all five of these ways are *equally likely*, but only 2 of the ways are on the first die, so the desired conditional probability is 2/5.

1b. If A is the event that the card with black on both sides was chosen, and B is the event that black appears facing up, then we have $P(A \cap B) = (1/2)(1) = 1/2$ and $P(B) = (1/2)(1) + (1/2)(1/2) = 3/4$, so $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{1/2}{3/4} = 2/3$.

Another method of solution is that there are 3 black sides that can appear, and all three of these ways are *equally likely*. Since 2 of these 3 are on the card with black on both sides, then the desired conditional probability is 2/3.

2abcd. In each of these problems, any of the 52 cards could appear in the position under discussion, and all 52 of these cards are *equally likely* to appear in that position, but only 4 of these 52 are queens, so the desired conditional probability is 4/52.

2e. If A is the event that the 19th card is a queen, and B is the event that the 1st card is a queen, then we have $P(A \cap B) = (4/52)(3/51)$ and $P(B) = (4/52)$, so $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{(4/52)(3/51)}{4/52} = 3/51$.

Another method is to note that, once we have seen that the 1st card is a queen, there are 51 other cards that could appear at the 19th position, and they are all *equally likely*, and 3 of them are queens, so the desired conditional probability is 3/51.

2f. If A is the event that the 19th card is a queen, and B is the event that the 1st and 7th cards are queens, then we have $P(A \cap B) = (4/52)(3/51)(2/50)$ and $P(B) = (4/52)(3/51)$, so $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{(4/52)(3/51)(2/50)}{(4/52)(3/15)} = 2/50$.

Another method is to note that, once we have seen that the 1st and 7th cards are queens, there are 50 other cards that could appear at the 19th position, and they are all *equally likely*, and 2 of them are queens, so the desired conditional probability is 2/50.

3. If A is the event that at least one value of 4 appears on the dice, and B is the event that the sum is 8 or larger, then we have $P(A \cap B) = 5/36$ and $P(B) = 15/36$, so $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{5/36}{15/36} = 5/15 = 1/3$.

Another method of solution is to note that there are 15 ways that the sum can be 8 or larger, and all 15 of these ways are *equally likely*, but only 5 of the ways will have at least one value of 4 on the dice, so the desired conditional probability is $5/15 = 1/3$.

4. We have $P(A_0 \mid B) = \frac{P(A_0\cap B)}{P(B)} = \frac{3/96}{6/96} = 3/6 = 1/2$, and $P(A_1 \mid B) = \frac{P(A_1\cap B)}{P(B)} = \frac{2/96}{6/96} = 2/6 = 1/3$, and $P(A_2 \mid B) = \frac{P(A_2\cap B)}{P(B)} = \frac{1/96}{6/96} = 1/6$. Indeed, these conditional probabilities do sum to 1.