STAT/MA 41600
In-Class Problem Set #40: November 26, 2018

1. Suppose that X and Y are random variables with joint probability mass function $p_{X,Y}(x,y) = (11/16)(1/4)^{x-1}(1/3)^{y-1}$ for integers $1 \leq y \leq x$.

Find $\mathbb{E}(X \mid Y = 1)$, i.e., find the conditional expectation of X, given that $Y = 1$.

2. Suppose that X and Y have a constant joint probability density function on the triangle with vertices at $(0,0)$, $(0,6)$, $(10,0)$, so that $f_{X,Y}(x,y) = 1/30$ for values (x,y) in this triangle, and $f_{X,Y}(x,y) = 0$ otherwise.

Find $\mathbb{E}(X \mid Y = 1)$, i.e., find the conditional expectation of X, given that $Y = 1$.

3. As in question 1 on Problem Set 26, suppose that the time (in seconds) until the next message arrives in Group Me is a continuous random variable X, and the time until the reply is denoted by Y. For this reason, we always have $Y > X$.

Suppose that the joint probability density function of X and Y is

$$f_{X,Y}(x,y) = \frac{1}{750} e^{-(x/150+y/30)}$$

for $y > x > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.

Given that $X = 20$, find the conditional expected value of Y.

4. Consider a 6-sided red die and a 6-sided blue die. Roll both of the dice. Given that the sum is exactly 9, find the conditional expected value of the value of the blue die.