STAT/MA 41600

In-Class Problem Set #39: November 12, 2018

1. As in question 1 on Problem Set 12, draw five cards from a deck with replacement (and reshuffling) in between the draws. Let X denote the number of cards with pictures of people (Jacks, Queens, and Kings) that appear. Let $X_i = 1$ if the ith card has a picture of a person (Jack, Queen, King), and $X_i = 0$ otherwise.

1a. Find $\mathrm{Cov}(X_i, X_i) = \mathbb{E}(X_i X_i) - \mathbb{E}(X_i)\mathbb{E}(X_i)$ for $1 \leq i \leq 5$.

1b. Find $\mathrm{Cov}(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j)$ for $1 \leq i \leq 5$ and $1 \leq j \leq 5$ with $i \neq j$.

1c. Use your work from 1a and 1b to obtain the variance of X (i.e., the covariance of X with itself). Does it agree with the solution to 1c and 1d on Problem Set 12?

2. As in question 2 on Problem Set 12, draw five cards from a deck, this time *without replacement*. Let X denote the number of cards with pictures of people (Jacks, Queens, and Kings) that appear. Let $X_i = 1$ if the ith card has a picture of a person (Jack, Queen, King), and $X_i = 0$ otherwise.

2a. Find $\mathrm{Cov}(X_i, X_i) = \mathbb{E}(X_i X_i) - \mathbb{E}(X_i)\mathbb{E}(X_i)$ for $1 \leq i \leq 5$.

2b. Find $\mathrm{Cov}(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j)$ for $1 \leq i \leq 5$ and $1 \leq j \leq 5$ with $i \neq j$.

2c. Use your work from 2a and 2b to obtain the variance of X (i.e., the covariance of X with itself). Does it agree with the solution to 2c on Problem Set 12?

3. As in question 3 on Problem Set 12, roll three 4-sided dice. Let X denote the minimum of the values that appear. Let $X_i = 1$ if the minimum is bigger than or equal to i, and $X_i = 0$ otherwise.

3a. Find $\mathrm{Cov}(X_i, X_i) = \mathbb{E}(X_i X_i) - \mathbb{E}(X_i)\mathbb{E}(X_i)$ for $1 \leq i \leq 4$.

3b. Find $\mathrm{Cov}(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j)$ for $1 \leq i \leq 4$ and $1 \leq j \leq 4$ with $i \neq j$.

3c. Use your work from 3a and 3b to obtain the variance of X (i.e., the covariance of X with itself). Does it agree with the solution to 3c on Problem Set 12?

4. Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. These 6 bears are all placed in a straight line, and all arrangements in such a line are equally likely. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs. Let $X_i = 1$ if the ith bear pair is sitting together, and $X_i = 0$ otherwise.

4a. Find $\mathrm{Cov}(X_i, X_i) = \mathbb{E}(X_i X_i) - \mathbb{E}(X_i)\mathbb{E}(X_i)$ for $1 \leq i \leq 3$.

4b. Find $\mathrm{Cov}(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j)$ for $1 \leq i \leq 3$ and $1 \leq j \leq 3$ with $i \neq j$.

4c. Use your work from 4a and 4b to obtain the variance of X (i.e., the covariance of X with itself). Does it agree with the solution to 4c on Problem Set 12?