STAT/MA 41600
In-Class Problem Set #34: November 2, 2018
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**Problem Set 34 Answers**

**1a.** The expected value is $\alpha/(\alpha+\beta) = 6/(6+2) = 6/8 = 3/4$.
**1b.** The probability density function is $f_X(x) = \frac{\Gamma(6+2)}{\Gamma(6)\Gamma(2)} x^{6-1}(1-x)^{2-1} = 42x^5(1-x)$ for $0 \le x \le 1$, and $f_X(x) = 0$ otherwise.
**1c.** We note $f_X(x) \ge 0$ for all $x$, and $\int_0^1 42x^5(1-x)\,dx = \int_0^1 42(x^5 - x^6)\,dx = 42(x^6/6 - x^7/7)|_{x=0}^1 = 42(1/6 - 1/7) = 42(7/42 - 6/42) = 1$.

**2.** We have $P(X < 1/4) = \int_0^{1/4} 42x^5(1-x)\,dx = \int_0^{1/4} 42(x^5 - x^6)\,dx = 42(x^6/6 - x^7/7)|_{x=0}^{1/4} = 42((1/4)^6/6 - (1/4)^7/7) = 11/8192 = 0.001343$.

**3.** We have $P(U > X) = \int_0^1\int_0^u (1)\frac{\Gamma(2+3)}{\Gamma(2)\Gamma(3)} x^{2-1}(1-x)^{3-1}\,dx\,du = \int_0^1\int_0^u 12(x - 2x^2 + x^3)\,dx\,du = \int_0^1 12(x^2/2 - 2x^3/3 + x^4/4)|_{x=0}^u\,du = \int_0^1 12(u^2/2 - 2u^3/3 + u^4/4)\,du = 12(u^3/6 - u^4/6 + u^5/20)|_{u=0}^1 = 3/5$.

**4.** We compute $P(X \ge Y) = \sum_{y=1}^\infty \sum_{x=y}^\infty (1/6)(5/6)^{x-1}(1/2)(1/2)^{y-1} = \sum_{y=1}^\infty (5/6)^{y-1}(1/2)(1/2)^{y-1} = \sum_{y=1}^\infty (1/2)(5/12)^{y-1} = (1/2)/(1-5/12) = 6/7$.