STAT/MA 41600
In-Class Problem Set #32 part 2: October 29, 2018

1. Generalize the previous problem set, question #1: Suppose that U is a continuous uniform random variable on $[0, a]$ (where $a > 0$ is fixed), and suppose that X is an exponential random variable with $\mathbb{E}(X) = 1/\lambda$ (where $\lambda > 0$ is fixed). If U and X are independent, find $P(X > U)$.

2. At a city park in Mishawaka, suppose that an ornithologist waits for the next bird to arrive. She wants to take an excellent picture of the bird. At a certain time of day, she believes that the interarrival times of the birds are independent exponential random variables, with mean of 10 seconds.

Let X_1 denote the time (in seconds) until the next bird's arrival. Let X_2 denote the additional time (in seconds) until the second bird's arrival. (So, altogether, $X_1 + X_2$ is the time until the second bird's arrival.) We are assuming that X_1 and X_2 are independent exponential random variables, each with mean 10, so $X_1 + X_2$ has mean 20. Of course, $X_1 + X_2$ is not an exponential random variable.

Find $P(X_1 + X_2 \leq 25)$.

3. In a busy newsroom, suppose that the times until the next phone ringing, next email arriving, or next computer beeping are independent exponential random variables, with respective means of 30 seconds, 20 seconds, and 15 seconds. Find the probability that the newsroom is silent for the next 10 seconds, i.e., that none of these events occur during the next 10 seconds.

4. Let X and Y be independent exponential random variables with $\mathbb{E}(X) = 3$ and $\mathbb{E}(Y) = 4$. Find $P(|X - Y| \leq 1)$.