STAT/MA 41600
In-Class Problem Set #27: October 17, 2018

1. Suppose that the time (in seconds) until the next message arrives in Group Me is a continuous random variable X, and the time until the reply is denoted by Y. For this reason, we always have $Y > X$.

Suppose that the joint probability density function of X and Y is

$$f_{X,Y}(x,y) = \frac{1}{750} e^{-(x/150+y/30)}$$

for $y > x > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.

Calculate $P(Y > 90 \mid X = 35)$.

2. Use X and Y from #1. Calculate $P(Y > 90 \mid X > 35)$.

3. Suppose that X and Y have a constant joint probability density function on the triangle with vertices at $(0,0)$, $(0,6)$, $(10,0)$, so that $f_{X,Y}(x,y) = 1/30$ for values (x,y) in this triangle, and $f_{X,Y}(x,y) = 0$ otherwise.
3a. Find $P(X > 3 \mid Y = 1)$.
3b. Find $P(X > 3 \mid Y > 1)$.

4. Suppose that, for some constant value k, the random variables X, Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} k(3-x)(4-y) & \text{for } 0 < x < 3 \text{ and } 0 < y < 4, \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(X + Y \leq 2)$.