STAT/MA 41600
In-Class Problem Set #24: October 10, 2018
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Problem Set 24 Answers

1a. We calculate $P(X > 20) = \int_{20}^{\infty} \frac{1}{25} e^{-x/25}\, dx = -e^{-x/25}|_{x=20}^{\infty} = e^{-20/25} = e^{-4/5} = 0.4493$.

1b. The probability that no Group Me message arrives in the next 1 minute (60 seconds) is $P(X > 60) = \int_{60}^{\infty} \frac{1}{25} e^{-x/25}\, dx = -e^{-x/25}|_{x=60}^{\infty} = e^{-60/25} = e^{-12/5} = 0.0907$.

1c. The median value "a" has the property $1/2 = P(X > a) = \int_{a}^{\infty} \frac{1}{25} e^{-x/25}\, dx = -e^{-x/25}|_{x=a}^{\infty} = e^{-a/25}$. Since we have $1/2 = e^{-a/25}$ then $\ln(1/2) = -a/25$, so the median is $a = -25 \ln(1/2)$.

2a. We have $f_X(x) = 1/6.2$ for $0 \le x \le 6.2 = 0.1613$.

2b. We have $P(X > 2) = \int_{2}^{6.2} 1/6.2\, dx = (6.2 - 2)/6.2 = 0.6774$.

2c. The median value "a" has the property $1/2 = P(X > a) = \int_{a}^{6.2} 1/6.2\, dx = (6.2 - a)/6.2$. Since we have $1/2 = (6.2 - a)/6.2$ then $(1/2)(6.2) = 6.2 - a$, and we conclude $a = 6.2 - (1/2)(6.2) = 3.1$.

3a. We calculate $P(28 < 2X < 65) = P(14 < X < 32.5) = \int_{14}^{32.5} \frac{1}{25} e^{-x/25}\, dx = -e^{-x/25}|_{x=14}^{32.5} = e^{-14/25} - e^{-32.5/25} = 0.2987$.

3b. We have $P(|X - 3.1| > 1.5) = P(X - 3.1 > 1.5) + P(X - 3.1 < -1.5) = P(X > 4.6) + P(X < 1.6) = \int_{4.6}^{6.2} 1/6.2\, dx + \int_{0}^{1.6} 1/6.2\, dx = (6.2 - 4.6)/6.2 + (1.6 - 0)/6.2 = 3.2/6.2 = 0.5161$.

4a. We have $1 = \int_{3}^{4} (k)(x-3)(x-4)\, dx = \int_{3}^{4} (k)(x^2 - 7x + 12)\, dx = (k)(x^3/3 - 7x^2/2 + 12x)|_{x=3}^{4} = (k)(40/3 - 27/2) = (k)(-1/6)$, and therefore, we conclude that $k = -6$.

4b. We compute $P(X > 3.25) = \int_{3.25}^{4} (-6)(x-3)(x-4)\, dx = \int_{3.25}^{4} (-6)(x^2 - 7x + 12)\, dx = (-6)(x^3/3 - 7x^2/2 + 12x)|_{3.25}^{4} = (-6)(40/3 - 2587/192) = 27/32 = 0.84375$.