STAT/MA 41600
In-Class Problem Set #22: October 1, 2018

1. At Earhart dining court, a new chef is cooking the stir-fry, and he has accidentally discarded the slips of paper on each plate. As a result, students cannot easily tell which plate is theirs. Chaos ensues! As a result, each student randomly picks a plate, and we might assume that all such selections are equally likely.
1a. If there are 8 such students, what is the expected number of students who (randomly) get their own proper plate of food?
1b. Again assuming that there are 8 such students, what is the variance of the number of students who (randomly) get their own proper plate of food?

2. Consider a discrete random variable X that is uniformly distributed between 1 and 5 (inclusive). Also consider a Geometric random variable Y with $\mathbb{E}(Y) = 3$, which is independent from X.
2a. Calculate $P(Y = X)$.
2b. Calculate $P(Y > X)$.

3. Same questions as **2ab**, but now assume X is a Poisson random variable with $\mathbb{E}(X) = 5$.

4. Consider 7 students and 7 available pencils (of 7 different colors) placed on a table at the front of the classroom.

Students are only happy when they get their favorite color of pencil. Each student only gets 1 pencil (without replacement).
4a. Suppose that all students prefer to get the blue pencil. What is the expected number of happy students?
4b. Suppose again that all students prefer to get the blue pencil. What is the variance of the number of happy students?
4cd. Repeat **4a** and **4b** but suppose (instead) that all students have unique color preferences.