STAT/MA 41600
In-Class Problem Set #16: September 21, 2018
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Problem Set 16 Answers

1. The number of interviews is a Geometric random variable with $p = .53$, so the expected value is $1/p = 1.8868$ and the variance is $q/p^2 = 1.6732$.

2. The number of rolls is a Geometric random variable with $p = 14/24 = 7/12$, so the expected number of rolls is $12/7 = 1.7143$.

3a. The probability that X is even is

$$\begin{aligned}
&P(X = 2) + P(X = 4) + P(X = 6) + P(X = 8) + P(X = 10) + \cdots \\
&= qp + q^3p + q^5p + q^7p + q^9p + \cdots \\
&= qp(1 + q^2 + (q^2)^2 + (q^2)^3 + (q^2)^4 + \cdots) \\
&= qp/(1 - q^2)
\end{aligned}$$

which simplifies to $qp/((1-q)(1+q)) = q/(1+q) = (1-p)/(2-p)$.

3b. The probability that X is a multiple of 3 is

$$\begin{aligned}
&P(X = 3) + P(X = 6) + P(X = 9) + P(X = 12) + P(X = 15) + \cdots \\
&= q^2p + q^5p + q^8p + q^{11}p + q^{14}p + \cdots \\
&= q^2p(1 + q^3 + (q^3)^2 + (q^3)^3 + (q^3)^4 + \cdots) \\
&= q^2p/(1 - q^3)
\end{aligned}$$

which simplifies to $q^2p/((1-q)(1+q+q^2)) = q^2/(1+q+q^2)$.

4. Method #1: We can ignore any green, yellow, or red bears that appear. In this new model, The total number of bears drawn is a Geometric random variable with $p = 2/3$, so the expected number of bears selected is $3/2$. So the expected number of bears that are not purple or orange is $3/2 - 1 = 1/2$, and in this model, these bears are all blue.

Method #2: The total number of bears drawn is a Geometric random variable with $p = 2/6$, so the expected number of bears selected is $6/2 = 3$. So the expected number of bears that are not purple or orange is $3 - 1 = 2$. Each of these bears is equally likely to be any of the four remaining colors, so the expected number of blue bears is $2/4 = 1/2$.