STAT/MA 41600
In-Class Problem Set #15: September 19, 2018
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Problem Set 15 Answers

1a. Yes, the random variable X is Binomial with parameters $n = 5$ and $p = 12/52 = 3/13$.
1b. No, the random variable X is not a Binomial random variable. The cards are selected without replacement, so the probability mass function does not have the form of $p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$.

2a. Since X is a Binomial random variable with $n = 3$ and $p = 2/3$, it follows that $\mathbb{E}(X) = np = 2$ and $\mathrm{Var}(X) = np(1-p) = (3)(2/3)(1/3) = 2/3$.
2b. We have $\mathbb{E}(Y) = \mathbb{E}(3X_1) = 3\mathbb{E}(X_1) = (3)(2/3) = 2$, and $\mathrm{Var}(Y) = \mathrm{Var}(3X_1) = 3^2\,\mathrm{Var}(X_1) = (3^2)(2/3)(1/3) = 2$.
2c. The random variables X and Y are dependent. For example, if $Y = 0$ then $X_1 = 0$ which makes $X \neq 3$. As another example, if $Y = 3$ then $X_1 = 1$ which makes $X \neq 0$.
2d. Yes, the random variable X has a Binomial distribution with $n = 3$ and $p = 2/3$.
2e. No, the random variable Y is not a Binomial random variable. It only assumes that possible values 0 and 3. So there is no value of n for which Y has possible values $0, \ldots, n$.
2f. We have $X = Y$ if and only if all three of the X_j's are 0 or all three of the X_j's are 1. Therefore we get $P(X = Y) = P(X_1 = X_2 = X_3 = 0) + P(X_1 = X_2 = X_3 = 1) = (1/3)^3 + (2/3)^3 = 1/27 + 8/27 = 9/27 = 1/3$.

3a. The random variable X does not have a Binomial distribution. Even though it can be written as the sum of Bernoulli random variables (as we have done on the most recent problem sets), those Bernoulli random variables are dependent. Also, we always have $1 \leq X \leq 4$, so there is no value of n for which X has possible values $0, \ldots, n$.
3b. The X_j's are dependent. For instance, if $X_4 = 1$, then $X \geq 4$, which means $X \geq 2$, which means $X_2 = 1$.

4. We could add $P(X = 2) + \cdots + P(X = 10)$ but it is easier to calculate the complement, namely, $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - (4/6)^{10} - (10)(2/6)(4/6)^9 = 17635/19683$.