STAT/MA 41600
In-Class Problem Set #12: September 17, 2018

1. Draw five cards from a deck with replacement (and reshuffling) in between the draws. Let X denote the number of cards with pictures of people (Jacks, Queens, and Kings) that appear.

1a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #10, question 1a.

1b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_5$, where the X_j's are **independent** indicators. Expand $(X_1 + \cdots + X_5)^2$ into 25 terms, where 20 of them will behave one way, and the other 5 will behave another way.

1c. Find $\mathrm{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.

1d. Find $\mathrm{Var}(X)$ in a different way, namely, using $\mathrm{Var}(X) = \mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_5)$ since the X_j's are independent in this problem.

2. Draw five cards from a deck, this time *without replacement*. Let X denote the number of cards with pictures of people (Jacks, Queens, and Kings) that appear.

2a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #10, question 2a.

2b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_5$, where the X_j's are **dependent** indicators. Expand $(X_1 + \cdots + X_5)^2$ into 25 terms, where 20 of them will behave one way, and the other 5 will behave another way.

2c. Find $\mathrm{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.

3. Roll three 4-sided dice. Let X denote the minimum of the values that appear.

3a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #7.

3b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2 + X_3 + X_4$, where the X_j's are **dependent** indicators. Expand $(X_1 + X_2 + X_3 + X_4)^2$ into 16 terms, which have various types of behaviors.

Hint: Using the formulation from Problem Set 11, on this problem we used $X_j = 1$ if the minimum is bigger than or equal to j, and $X_j = 0$ otherwise. So on this problem, $X_i X_j = X_j$ if $j > i$. For example, $X_2 X_3 = X_3$ since $X_2 X_3 = 1$ if the minimum is at least 3, and $X_2 X_3 = 0$ otherwise. This problem might seem a little tricky at first, but just think about it, and discuss it, and make sure that your solution agrees with **3a**.

3c. Find $\mathrm{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.

4. Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. These 6 bears are all placed in a straight line, and all arrangements in such a line are equally likely. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs.

4a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #8.

4b. Find $\mathbb{E}(X^2)$ in a different way, namely, using a sum of indicator random variables, as in Problem Set #11.

4c. Find $\mathrm{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.