STAT/MA 41600
In-Class Problem Set #11: September 14, 2018

Solve all problems by decomposing the random variable in each problem as a sum of indicator random variables. In other words, find indicator random variables $X_1, X_2, X_3, \ldots$ such that $X = X_1 + X_2 + X_3 + \cdots$, and use the fact that $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3 + \cdots) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) + \cdots$ to find the expected value.

1. Draw five cards from a deck with replacement (and reshuffling) in between the draws. Let X denote the number of cards with pictures of people (Jacks, Queens, and Kings) that appear.

Define five indicator random variables so that $X = X_1 + \cdots + X_5$. Calculate the expected value of X as the sum of the expected values of these five indicator random variables.

2. Draw five cards from a deck, this time *without replacement*. Let X denote the number of cards with pictures of people (Jacks, Queens, and Kings) that appear.

Define five indicator random variables so that $X = X_1 + \cdots + X_5$. Calculate the expected value of X as the sum of the expected values of these five indicator random variables.

3. Roll three 4-sided dice. Let X denote the minimum of the values that appear.

Define four indicator random variables so that $X = X_1 + X_2 + X_3 + X_4$. Calculate the expected value of X as the sum of the expected values of these four indicator random variables.

4. Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. These 6 bears are all placed in a straight line, and all arrangements in such a line are equally likely. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs.

Define some indicator random variables so that X is the sum of these indicator random variables. Calculate the expected value of X as the sum of the expected values of these indicator random variables.

(There are several ways to setup question #4. You might enjoy setting it up in several different ways.)