STAT/MA 41600
In-Class Problem Set #10: September 12, 2018
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Problem Set 10 Answers

1a. We compute $P(X = 0) = (40/52)^5$, $P(X = 1) = (5)(40/52)^4(12/52)$, $P(X = 2) = (10)(40/52)^3(12/52)^2$, $P(X = 3) = (10)(40/52)^2(12/52)^3$, $P(X = 4) = (5)(40/52)(12/52)^4$, and $P(X = 5) = (12/52)^5$.
1b. It follows that $\mathbb{E}(X) = (0)(40/52)^5 + (1)(5)(40/52)^4(12/52) + (2)(10)(40/52)^3(12/52)^2 + (3)(10)(40/52)^2(12/52)^3 + (4)(5)(40/52)(12/52)^4 + (5)(12/52)^5 = 15/13$.

2a. We compute

$$P(X = 0) = (40/52)(39/51)(38/50)(37/49)(36/48) = 2109/8330$$
$$P(X = 1) = (5)(12/52)(40/51)(39/50)(38/49)(37/48) = 703/1666$$
$$P(X = 2) = (10)(12/52)(11/51)(40/50)(39/49)(38/48) = 209/833$$
$$P(X = 3) = (10)(12/52)(11/51)(10/50)(40/49)(39/48) = 55/833$$
$$P(X = 4) = (5)(12/52)(11/51)(10/50)(9/49)(40/48) = 165/21658$$
$$P(X = 5) = (12/52)(11/51)(10/50)(9/49)(8/48) = 33/108290$$

2b. It follows that $\mathbb{E}(X) = (0)(2109/8330) + (1)(703/1666) + (2)(209/833) + (3)(55/833) + (4)(165/21658) + (5)(33/108290) = 15/13$.

3. We recall that $P(X = 4) = (1/4)^3 = 1/64$, $P(X = 3) = (2/4)^3 - (1/4)^3 = 7/64$, $P(X = 2) = (3/4)^3 - (2/4)^3 = 19/64$, and $P(X = 1) = (4/4)^3 - (3/4)^3 = 37/64$. So we get $\mathbb{E}(X) = (4)(1/64) + (3)(7/64) + (2)(19/64) + (1)(37/64) = 25/16$.

4. We recall that $p_X(3) = 1/15$, $p_X(2) = 1/5$, $p_X(1) = 2/5$, and $p_X(0) = 1/3$. So we get $\mathbb{E}(X) = (3)(1/15) + (2)(1/5) + (1)(2/5) + (0)(1/3) = 1$.